MATHEMATICAL MODELLING AND ANALYSIS OF MULTIPHASE SYSTEMS IN PHASOR COORDINATES

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By
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CERTIFICATE

Certified that the thesis entitled 'MATHEMATICAL MODELLING AND ANALYSIS OF MULTIPHASE SYSTEMS IN PHASOR COORDINATES' by Mr. Sanjiv Swarup has been carried out under my supervision and this has not been submitted elsewhere for a degree.

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ABSTRACT

For a long time now, the analysis of a three phase power system network is being done on the assumption of a balanced system. However, due to certain inherent unbalances present in the system, it is necessary to evaluate the performance of the system in phase frame of reference. Moreover, the continuous increase in demand for power has exposed the limitations of three phase power systems. This has led to the feasibility studies of multiphase systems. In the present work, the detailed modelling of different elements present in three phase and multiphase (six phase and twelve phase) systems has been done. The modelling has been employed to carry out the balanced as well as unbalanced load flow analysi of three phase and six phase systems.

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CHAPTER 1

INTRODUCTION

With the ever increasing demand for electric power and consequently, with the growing size and complexities of the modern power systems, it has become necessary to search for some viable alternatives to meet such a demand which requires transfer of more and more power from remote power plants to far away load centres. One of the alternatives, being utilized at present, is the high voltage transmission. However, transmission at such high voltages is not free of demerits. It poses some problems, for example, strong electric field at the ground surface with possible biological effects, high audible and radio noise, visual pollution, increasingly difficult problems in acquiring new rights of way, etc. Although work is being done to tackle such problems through improved design methods, search for other alternatives to transfer more power is also going on.

One of the most promising alternatives to meet the increasing demand of electric power is the use of high phase order or multiphase systems, especially multiphase transmission. The idea of multiphase systems was first introduced by Barnes and Barthold in 1972 and ever since this topic has been an area of active research for power system engineers

and scientists. Feasibility studies of multiphase systems have shown that they offer considerable advantages over their three phase counterparts in several aspects.

It is essential to develop an accurate model of multiphase systems in order to study their feasibility. This is achieved by representing the different components of multiphase systems in phasor coordinates with the help of mathematical equations. The representation thus obtained is known as mathematical modelling of the multiphase systems which is a simple and inexpensive means to analyse the systems. The mathematical model, thus developed, is used to carry out the analysis of multiphase systems such as load flow analysis, fault analysis and transient stability analysis, etc.

1.1 HISTORICAL SURVEY

The concept of multiphase transmission was first introduced by Barnes and Barthold in 1972 [1]. This led J.R. Stewart et al [2,3] to carry out the feasibility studies of multiphase systems based on steady state considerations and their overvoltages and insulation requirements. Venkata et al [5,6] pursued the topic by conducting preliminary investigation of power transfer capability, load flow, stability and reliability aspects of six phase systems. Grant et al [4] have confirmed analytical predictions of electrical and mechanical behaviour of multiphase systems based on the studies of six

and twelve phase test systems. Laughton et al [7,12,15,16] have developed mathematical models of three phase transformers and machines in the phase frame of reference and have conducted the load flow and fault analysis of three phase systems. Roy et al [17] have also carried out the load flow analysis of three phase systems. The usual short circuit analysis of six phase transmission system was presented by Bhatt et al [18] and Nanda et al [19]. Venkata et al [21] have discussed the various types of faults and their significant combinations likely to occur on a six phase line. The analytical expressions for all the 23 significant faults have been developed. The effect of converting a double circuit three-phase line to a six-phase line was studied by L.P. Singh et al [10]. Furhter, theoretical work in this field such as modelling, fault analysis, etc. of six phase systems has been taken up by S.N. Tewari et al [8-11]. Uma Pal et al [24] have reported on feasibility of twelve phase systems and their fault analysis. The symmetrical component transformation for six phase systems has been derived by Bhatt et al [18] as a direct extension of the three phase technique. L.P. Singh et al [22,23] have derived power invariant symmetrical component transformations for six and higher phase order systems using group theoretic techniques. L. Roy [20] has given an algorithm for developing the admittance matrix of polyphase systems. This is utilized to carry out the load flow and fault analysis of multiphase systems in phasor coordinates.

All these studies on multiphase systems have encouraged electrical engineers and scientists to pursue the topic in more detail in order to establish the superiority of multiphase systems, especially the six phase systems, over the conventional three phase systems.

In the present work, the detailed representation of three and multiphase (six and twelve-phase) systems is done in phasor coordinates. This helps in taking into account various types of unbalances existing in the system which are, otherwise, left untouched. The advantage of such a representation is, that, it retains the physical identities of the different elements present in the systems. Models to analyse composite three-phase and six-phase systems and composite three-phase and twelve-phase systems have been derived. The representation is used to carry out the balanced as well as the unbalanced load flow analysis of three and six-phase systems. We now give the chapterwise description of the work carried out in this thesis.

1.2 CHAPTERWISE DESCRIPTION

Chapter 2 starts with the emphasis on representing the three-phase system in phasor coordinates. The detailed modelling of three-phase elements, such as machines, different types of transformers, transmission lines has been given using the symmetrical lattice equivalent circuits. This modelling is employed to carry out the balanced as well as unbalanced load flow analysis of a sample power system.

Chapter 3 initially presents an overview of the possibility of multiphase transmission systems from steady state and overvoltages and insulation requirements point of view. This is followed by mathematical modelling of the different components such as synchronous machines, transmission lines and transformers of six.phase systems. The transmission line has been modelled based upon its phase impedance matrix, ABCD parameters and π-equivalent circuit. Three-phase equivalent representation of a six-phase system and vice versa has been derived. This equivalent representation is utilised to carry out the load flow studies of a sample power system on equivalent three phase basis. The chapter concludes with the load flow analysis of a composite three and six-phase power system.

Chapter 4 starts with the development of detailed mathematical model of machines, transformers and transmission lines of twelve-phase systems in phasor coordinates. Phase impedance matrix representation, ABCD parameter representation and the π-equivalent representation of the twelve phase transmission line has been given. The three-phase equivalent representation of a twelve-phase system and vice versa have also been developed.

Chapter 5 summarises the work reported in this thesis and the conclusions drawn from the results and findings of the work.

CHAPTER 2

THREE PHASE POWER SYSTEM: MODELLING AND ANALYSIS

2.1 INTRODUCTION

Traditionally the analysis of power systems is done based on the assumption of a balanced system. This is done by representing the system by a one line diagram. Although such an analysis gives sufficiently accurate results, it fails to take into account the inherent unbalances present in the system. There are a number of situations in practical power systems like very long untransposed transmission lines, bundle conductors, large single phase loads, one, two or four phase supply, single pole switching, etc. which make the system unbalanced and warrant the need for a more detailed representation of the system.

From the analysis of a balanced system, various parameters namely, voltage magnitude, its angle, current, real and reactive powers at a bus are found for one phase only and those for the other two phases are obtained from the concepts of a balanced system. However, in an unbalanced system, such a procedure cannot be employed because of the obvious reasons.

With a view to analysing the power systems taking into account the unbalances present in it, it becomes imperative to

represent the system in detail. This requires a critical modelling of the various elements of a power system such as machines, transformers, transmission lines and loads, etc.

This chapter deals exclusively with the modelling of three phase power systems. The chapter starts with the derivation of the final equation of a machine representing all the unbalances in it. After this the representation of different types of transformers are given based on their symmetrical lattice equivalent circuits. Finally the modelling of transmission lines and loads are done. This modelling of the system is employed to carry out the three phase load flow studies of a sample system in phasor coordinates.

2.2 MODELLING OF THREE PHASE MACHINES

The general three phase element can be represented as shown in Fig. 2.1. Fig. 2.1 represents a wye-connected machine if E_a, E_b, E_c are a balanced set of internal voltages and nodes 4,5 and 6 are joined to form a star point [15]. Hence, a three phase synchronous generator is represented by a balanced set of internal voltages behind proper reactances. Fig. 2.2 shows the self and mutual admittances for any node a in detail.

The KCL at any node a is written as

$$\left\{(V_4+E_a)-V_1\right\} Y_{aa}+\left\{(V_4+E_a)-V_2\right\}(-Y_{ab})+\left\{(V_4+E_a)-V_3\right\}(-Y_{ac})$$
$$+\left\{(V_4+E_a)-(V_5+E_b)\right\} Y_{ab}+\left\{(V_4+E_a)-(V_6+E_c)\right\} Y_{ac}=I_4 \quad (2.1)$$

or,

$$\begin{bmatrix} -Y_{aa} & Y_{ab} & Y_{ac} & Y_{aa} & -Y_{ab} & -Y_{ac} & Y_{aa} & -Y_{ab} & -Y_{ac} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \\ E_{abc} \end{bmatrix} = \begin{bmatrix} I_4 \end{bmatrix} \tag{2.2}$$

where

$$V_{123} = [V_1 \ V_2 \ V_3]^T$$

$$V_{456} = [V_4 \ V_5 \ V_6]^T$$

$$E_{abc} = [E_a \ E_b \ E_c]^T$$

Similarly, from Fig. 2.2, we can write equations at nodes b and c also. Combining all these equations, we obtain the expression of the following form :

$$\begin{bmatrix} -Y_{aa} & Y_{ab} & Y_{ac} & Y_{aa} & -Y_{ab} & -Y_{ac} & Y_{aa} & -Y_{ab} & -Y_{ac} \\ Y_{ba} & -Y_{bb} & Y_{bc} & -Y_{ba} & Y_{bb} & -Y_{bc} & -Y_{ba} & Y_{bb} & -Y_{bc} \\ Y_{ca} & Y_{cb} & -Y_{cc} & -Y_{ca} & -Y_{cb} & Y_{cc} & -Y_{ca} & -Y_{cb} & Y_{cc} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \\ E_{abc} \end{bmatrix}$$

$$= \begin{bmatrix} I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{2.3}$$

Likewise, Fig. 2.3 shows the admittances at node 1 . Thus KCL at node 1 is :

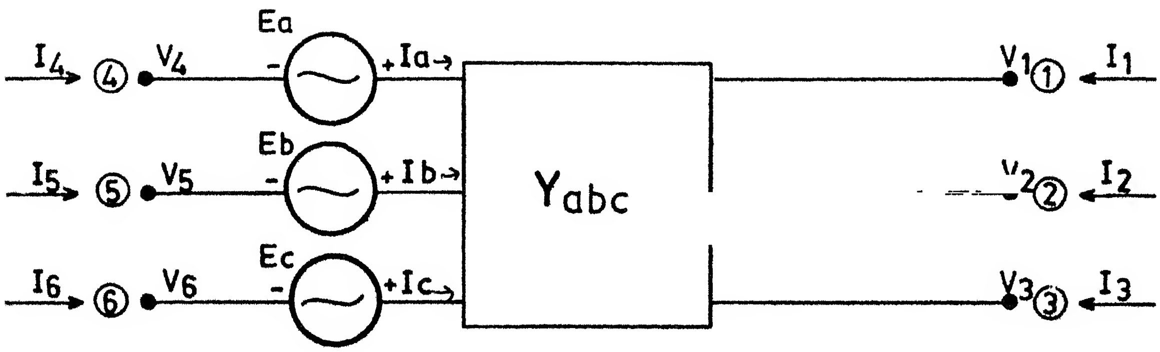


FIG. 2.1 SCHEMATIC REPRESENTATION OF A THREE PHASE ELEMENT

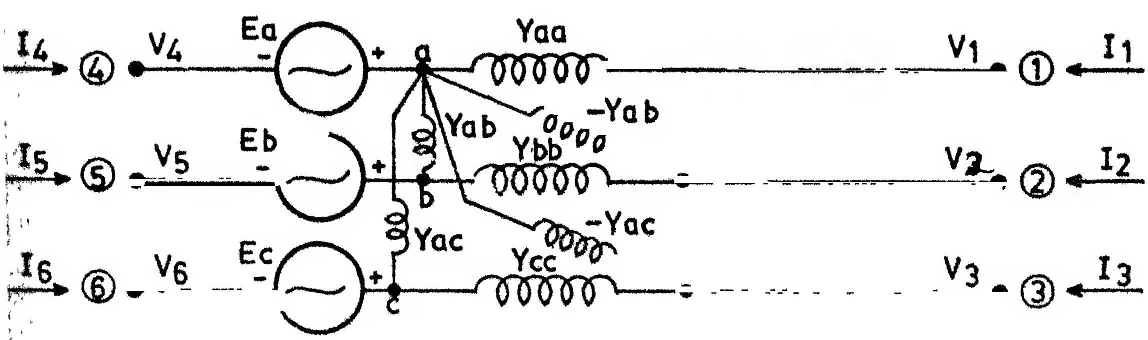


FIG. 2.2 SCHEMATIC REPRESENTATION OF SELF AND MUTUAL ADMITTANCES AT ANY NODE A OF A THREE PHASE MACHINE.

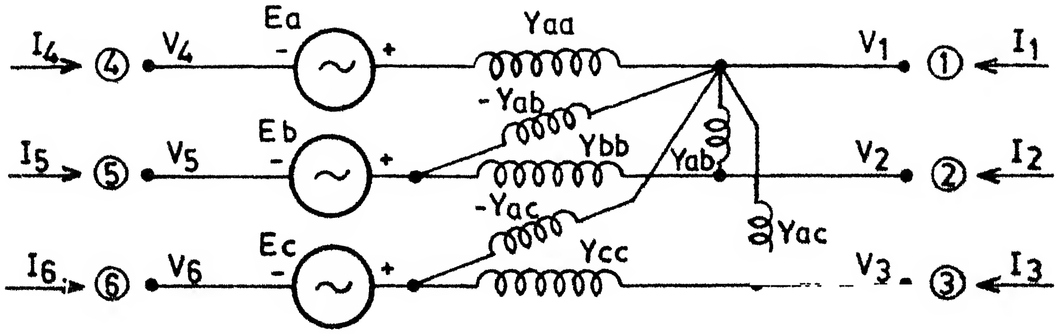


FIG. 2.3 SCHEMATIC REPRESENTATION OF SELF AND MUTUAL ADMITTANCES AT ANY NODE 1 OF A 3-Ø MACHINE

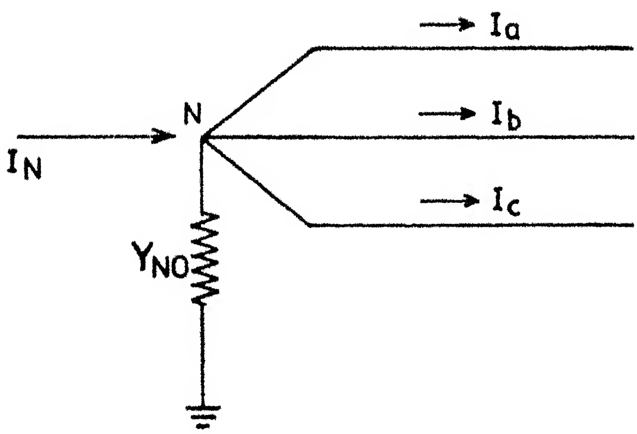


FIG. 2.4 THE INJECTED CURRENT AT THE NEUTRAL OF A THREE PHASE GENERATOR.

$$\left\{V_1-(V_4+E_a)\right\} Y_{aa}+\left\{V_1-(V_5+E_b)\right\}(-Y_{ab})+\left\{V_1-(V_6+E_c)\right\}(-Y_{ac})$$

$$+ (V_1-V_2)Y_{ab}+(V_1-V_3)Y_{ac} = I_1 \qquad (2.4)$$

or

$$\begin{bmatrix} Y_{aa} & -Y_{ab} & -Y_{ac} & -Y_{aa} & Y_{ab} & Y_{ac} & -Y_{aa} & Y_{ab} & Y_{ac} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \\ E_{abc} \end{bmatrix} = I_1 \qquad (2.5)$$

Proceeding in the same fashion, we can form equations similar to eqn. (2.3) by writing KCL at nodes 2 and 3. Combining these three equations results in the following equation :

$$\begin{bmatrix} Y_{aa} & -Y_{ab} & -Y_{ac} & -Y_{aa} & Y_{ab} & Y_{ac} & -Y_{aa} & Y_{ab} & Y_{ac} \\ -Y_{ba} & Y_{bb} & -Y_{bc} & Y_{ba} & -Y_{bb} & Y_{bc} & Y_{ba} & -Y_{bb} & Y_{bc} \\ -Y_{ca} & -Y_{cb} & Y_{cc} & Y_{ca} & Y_{cb} & -Y_{cc} & Y_{ca} & Y_{cb} & -Y_{cc} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \\ E_{abc} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad (2.6)$$

Combining eqns. (2.3) and (2.6), we get

$$\begin{bmatrix} Y_{abc} & -Y_{abc} & -Y_{abc} \\ -Y_{abc} & Y_{abc} & Y_{abc} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \\ E_{abc} \end{bmatrix} = \begin{bmatrix} I_{123} \\ I_{456} \end{bmatrix} \qquad (2.7)$$

where,

$$Y_{abc} = \begin{bmatrix} Y_{aa} & -Y_{ab} & -Y_{ac} \\ -Y_{ba} & Y_{bb} & -Y_{bc} \\ -Y_{ca} & -Y_{cb} & Y_{cc} \end{bmatrix} \qquad (2.8)$$

$$I_{123} = [I_1 \quad I_2 \quad I_3]^T \quad ,$$

and

$$I_{456} = [I_4 \quad I_5 \quad I_6]^T$$

Eqn. (2.7) is the concise form of nodal relationships for the current and voltage reference directions shown in Fig. 2.1.

If the parameters of machine are given in the component form, the phase admittance matrix Y_{abc} is obtained by

$$Y_{abc} = T_s \; Y^{0,1,2} \; T_s^{*T} \tag{2.9}$$

when

$$T_s = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$\alpha = 3\sqrt{1} = 1\underline{/120^\circ} = e^{j2\pi/3} = -0.5+j0.866$

and

$$Y^{0,1,2} = \text{diag.}\ [y_o, y_1, y_2]$$

After using eqn. (2.9), we obtain

$$Y_{abc} = \frac{1}{3} \begin{bmatrix} (y_o+y_1+y_2) & (y_o+\alpha y_1+\alpha^2 y_2) & (y_o+\alpha^2 y_1+\alpha y_2) \\ (y_o+\alpha^2 y_1+\alpha y_2) & (y_o+y_1+y_2) & (y_o+\alpha y_1+\alpha^2 y_2) \\ (y_o+\alpha y_1+\alpha^2 y_2) & (y_o+\alpha^2 y_1+\alpha y_2) & (y_o+y_1+y_2) \end{bmatrix} \tag{2.10}$$

where y_o, y_1 and y_2 are the zero, positive and negative sequence admittances of the synchronous generator respectively. For balanced machine interval voltages,

$$E_b = \alpha^2 E_a \quad \text{and} \quad E_c = \alpha\, E_a$$

If in Fig. 2.1, nodes 4,5 and 6 are joined to form a neutral point N, then we have

$$I_N = I_4 + I_5 + I_6 \tag{2.11}$$

and

$$V_N = V_4 = V_5 = V_6 \tag{2.12}$$

where I_N is the injected current at the neutral N and V_N is the neutral voltage.

If S_a, S_b and S_c denote the complex powers in phases a,b and c respectively, then

V_N+E_a, V_N+E_b and V_N+E_c are the interval terminal voltages and

$$I_a = \frac{S_a^*}{(V_N+E_a)^*} \; ; \; I_b = \frac{S_b^*}{(V_N+E_b)^*} \; ; \; I_c = \frac{S_c^*}{(V_N+E_c)^*} \tag{2.13}$$

Substituting the value of Y_{abc} from eqn. (2.10) into eqn. (2.7) and also the values of E_b and E_c and taking

$$I_4 = I_a, \; I_5 = I_b, \; I_6 = I_c,$$

eqn. (2.7), after collection of common terms for V_N and E_a, reduces to the following :

$$\left[\begin{array}{c|cc} & -y_o & -y_1 \\ Y_{abc} & -y_o & -\alpha^2 y_1 \\ & -y_o & -\alpha y_1 \\ \hline & y_o & y_1 \\ -Y_{abc} & y_o & \alpha^2 y_1 \\ & y_o & \alpha y_1 \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline V_N \\ E_a \end{array}\right] = \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ \hline S_a^*/(V_N+E_a)^* \\ S_b^*/(V_N+E_b)^* \\ S_c^*/(V_N+E_c)^* \end{array}\right] \qquad (2.14)$$

From the eqn. (2.11), we note that an additional eqn. can be written for node N. Hence, after proper substitution from eqn. (2.10) in eqn. (2.14) and adding last three equations of (2.14) gives :

$$[-y_o \quad -y_o \quad -y_o \quad 3y_o \quad 0] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_N \\ E_a \end{bmatrix} = [I_N] \qquad (2.15)$$

Noting that $E_b^* = \alpha E_a^*$ and $E_c^* = \alpha^2 E_a^*$, the last three right hand side terms of eqn. (2.14) become

$$\frac{S_a^*}{V_N^*+E_a^*} \; ; \; \frac{S_b^*}{V_N^*+\alpha E_a^*} \quad \text{and} \quad \frac{S_c^*}{V_N^*+\alpha^2 E_a^*} \quad \text{respectively.}$$

Cross multiplying the last three rows of eqn. (2.14), we obtain

$$\frac{1}{3}\,[-(y_o+y_1+y_2)\;V_1(V_N^*+E_a^*)-(y_o+\alpha y_1+\alpha^2y_2)V_2(V_N^*+E_a^*)$$

$$-(y_o+\alpha^2y_1+\alpha y_2)V_3(V_N^*+E_a^*)]+\;y_oV_N(V_N^*+E_a^*)+y_1E_a(V_N^*+E_a^*)\;=\;S_a^* \tag{2.17}$$

$$\frac{1}{3}\,[-(y_o+\alpha^2y_1+\alpha y_2)\;V_1(V_N^*+\alpha E_a^*)-(y_o+y_1+y_2)V_2(V_N^*+\alpha E_a^*)$$

$$-(y_o+\alpha y_1+\alpha^2y_2)V_3(V_N^*+\alpha E_a^*)]+y_oV_N(V_N^*+\alpha E_a^*)+\alpha^2y_1E_a(V_N^*+\alpha E_a^*)\;=\;S_b^* \tag{2.18}$$

$$\frac{1}{3}\,[-(y_o+\alpha y_1+\alpha^2y_2)V_1(V_N^*+\alpha^2E_a^*)-(y_o+\alpha^2y_1+\alpha y_2)V_2(V_N^*+\alpha^2E_a^*)$$

$$-(y_o+y_1+y_2)V_3(V_N^*+\alpha^2E_a^*)]+y_oV_N(V_N^*+\alpha^2E_a^*)+\alpha y_1E_a(V_N^*+\alpha^2E_a^*)\;=\;S_c^* \tag{2.19}$$

Adding eqns. (2.17), (2.18) and (2.19) and simplifying we obtain

$$-y_oV_1V_N^*-y_oV_2V_N^*-y_oV_3V_N^*+3y_oV_NV_N^*-y_1V_1E_a^*$$

$$-\alpha y_1V_2E_a^*-\alpha^2y_1V_3E_a^*+3y_1E_aE_a^*\;=\;S_a^*+S_b^*+S_c^* \tag{2.20}$$

The first four terms of eqn . (2.20) are V_N^* times I_N. Hence, under usual conditions, $I_N = 0$ and eqn. (2.20) reduces to

$$[-y_1 \ -\alpha y_1 \ -\alpha^2 y_1 \ 3y_1] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ E_a \end{bmatrix} = \frac{S_a^* + S_b^* + S_c^*}{E_a^*} \tag{2.21}$$

Suppose the generator is earthed through an admittance y_{NO} as shown in Fig. 2.4, then

$$I_N = I_4 + I_5 + I_6 + y_{NO} V_N \tag{2.22}$$

$$= [-y_o \ -y_o \ -y_o \ 3y_o + y_{NO} \ \ 0] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_N \\ E_a \end{bmatrix} \tag{2.23}$$

Substituting the value of $V_N^* I_N$ in eqn. (2.23) and putting $I_N = 0$, we get

$$[-y_1 \ -\alpha y_1 \ -\alpha^2 y_1 \ 3y_1] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ E_a \end{bmatrix} = \frac{S_a^* + S_b^* + S_c^*}{E_a^*} + \frac{y_{NO} |V_N|^2}{E_a^*} \tag{2.24}$$

The second term on the R.H.S. of the eqn. (2.24) is negligible as compared to the first term. Hence, the final machine equation, with $I_N = 0$, is the combination of the first three eqns. of (2.14), eqn. (2.23) and eqn. (2.24) :

$$\left[\begin{array}{ccc|cc} & & & -y_1 & -y_o \\ & Y_{abc} & & -\alpha^2 y_1 & -y_o \\ & & & -\alpha y_1 & -y_o \\ \hline -y_1 & -\alpha y_1 & -\alpha^2 y_1 & 3y_1 & 0 \\ \hline -y_o & -y_o & -y & 0 & 3y_o+y_{NO} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hline E_a \\ \hline V_N \end{array}\right] = \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ \hline \Sigma S^*/E_a^* \\ \hline 0 \end{array}\right] \quad (2.25)$$

It may be noted that the machine model eqn. (2.25) makes use of the total power and not the individual phase powers. The injected nodal currents I_1, I_2, I_3 are zero for the synchronous generator if there is no local load. However, if there is a local load, they are not zero and one third of the total local load is assigned to each of the phases.

2.3 MODELLING OF THREE PHASE TRANSFORMERS

In this section, we shall first derive the symmetrical lattice equivalent circuit of a single phase transformer with off-nominal tappings on primary as well as on secondary sides. This lattice equivalent circuit will then be employed to represent different types of three phase transformers [7].

2.3.1 Symmetrical Lattice Equivalent of one-phase Transformers

Fig. 2.5 illustrates a single phase representation of a transformer in per unit form as an ideal transformer of turns

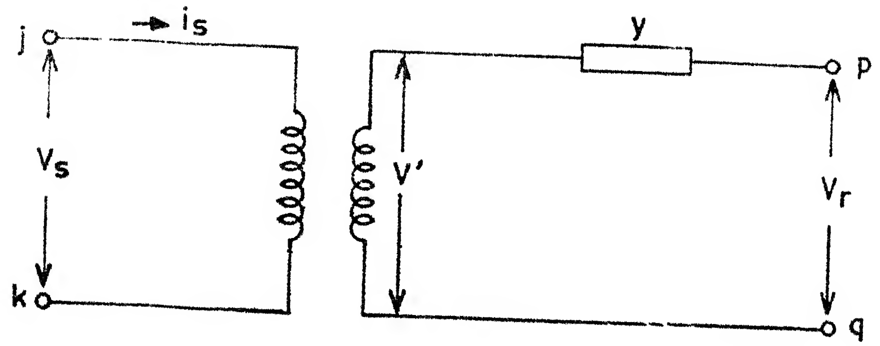


FIG. 2.5 SCHEMATIC REPRESENTATION OF AN IDEAL SINGLE PHASE TRANSFORMER.

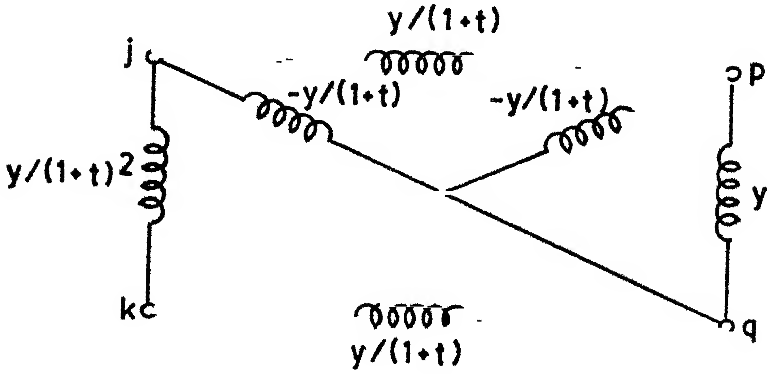


FIG. 2.6 SYMMETRICAL LATTICE EQUIVALENT CIRCUIT OF FIG. 2.5.

ratio (1+t):1 in series with an equivalent leakage admittance y. From Fig. (2.5) and the relationships in an ideal transformer, we can write

$$\begin{aligned} V_s &= (1+t)V' \\ i_r &= -(1+t)i_s \end{aligned} \tag{2.26}$$

The terminal relationships are

$$\begin{aligned} i_r &= y(V_r - V') = yV_r - \frac{yV_s}{1+t} \\ i_s &= -\frac{i_r}{1+t} = \frac{yV_s}{(1+t)^2} - \frac{yV_r}{(1+t)} \end{aligned} \tag{2.27}$$

If we do not assume any earth connections and represent the nodes of the transformer by j,k,p,q, then

$$V_s = V_j - V_k \quad \text{and} \quad V_r = V_p - V_q \tag{2.28}$$

The injected currents into each node for an n-node network are expressed in terms of these nodal voltages by equations of the form

$$\begin{aligned} I_j &= \sum_m I_{jm} + i_s \\ &= \sum_m (V_j - V_m) y_{jm} + \frac{y}{(1+t)^2}(V_j - V_k) - \frac{y}{(1+t)}(V_p - V_q) \end{aligned} \tag{2.29}$$

where the summation is over the set of all nodes m connected to node j excluding the set k,p,q. Expanding the eqn. (2.29) for each node of transformer, we obtain

$$I_j = (\sum_m y_{jm} + \frac{y}{(1+t)^2})V_j - \frac{y}{(1+t)^2} V_k - \frac{y}{(1+t)} V_p$$

$$+ \frac{y}{(1+t)} V_q + \sum_m (-y_{jm})V_m \tag{2.30}$$

$$I_k = - \frac{y}{(1+t)^2} V_j + (\sum_m y_{km} + \frac{y}{(1+t)^2})V_k + \frac{y}{(1+t)} V_p$$

$$- \frac{y}{(1+t)} V_q + \Sigma (-y_{km})V_m \tag{2.31}$$

$$I_p = - \frac{y}{(1+t)} V_j + \frac{y}{(1+t)} V_k + (\sum_m y_{pm} + y)V_p - yV_q + \Sigma(-y_{pm})V_m \tag{2.32}$$

$$I_q = \frac{y}{(1+t)} V_j - \frac{y}{(1+t)} V_k - yV_p + (\sum_m y_{qm} + y)V_q + \sum_m (-y_{qm})V_m \tag{2.33}$$

The nodal admittance relationships as seen from eqn. (2.30) to eqn. (2.33) may be written in the form

$$I = YV + Y'V \tag{2.34}$$

The equivalent circuit shown in Fig. 2.6 is deduced from the algorithm of forming matrix Y (by inspection). Thus it represents a single phase tapped transformer. The voltage and

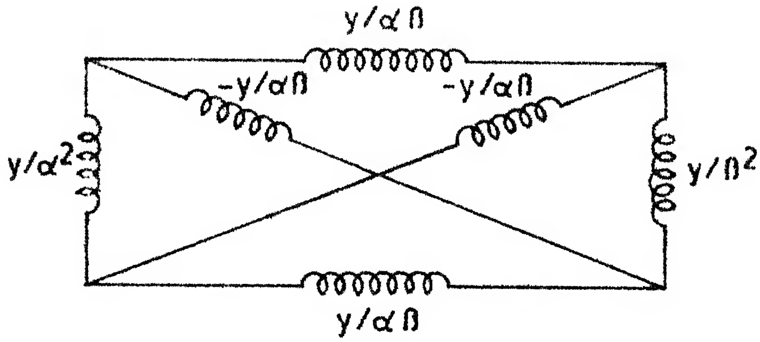


FIG. 2.7 GENERAL SYMMETRICAL LATTICE EQUIVALENT CIRCUIT OF A SINGLE PHASE TRANSFORMER.

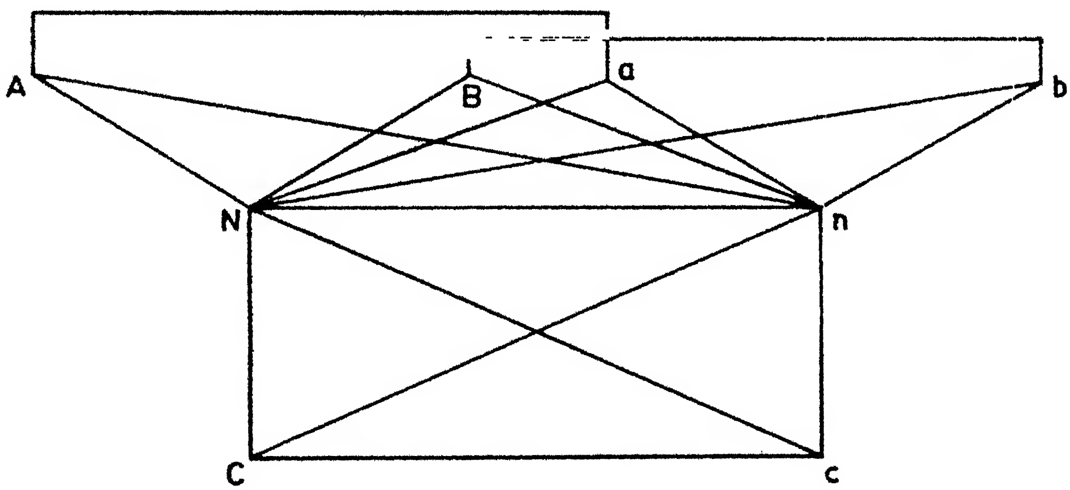


FIG. 2.8 SYMMETRICAL LATTICE EQUIVALENT CIRCUIT OF A 3-Ø STAR-STAR TRANSFORMER AS A LINEAR GRAPH.

current relationships between the nodes of this symmetrical lattice equivalent circuit are same as those in Fig. 2.5.

From Fig. 2.6 we can easily obtain the circuit shown in Fig. 2.7 [7] which represents the general symmetrical lattice equivalent circuit of a single phase transformer, where both primary and secondary windings may have either actual or equivalent variable turns ratio α and β or both.

2.3.2 Representation of Three-Phase Transformers

The general symmetrical lattice equivalent circuit derived in the Section 2.3.1 is employed to represent the equivalent circuit of different types of three phase transformers, namely, star-star, delta-delta and star-delta in this section.

a) Star-Star Transformer :

The lattice equivalent circuit of a three phase star-star transformer as a linear graph is shown in Fig. 2.8 in which parallel transformer windings are taken to represent the equivalent single phase transformers. The circuit is drawn in analogy with that of Fig. 2.7 with taps on only one (primary) side since in practice either α or β would be 1 p.u. The equivalent circuit may be described by the connection Table 2.1.

b) Delta-Delta Transformer :

The lattice equivalent circuit of the three phase delta-delta transformer as a linear graph with the same convention

Table 2.1

Connection table for Y-Y transformer with

$\alpha = 1+t_\alpha,\ \beta = 1$ p.u.

Admittances	Between Nodes
y/α^2	N-A, N-B, N-C
y	n-a, n-b, n-c
y/α	A-a, B-b, C-c
$-y/\alpha$	n-A, n-B, n-C; N-a, N-b, N-c
$3y/\alpha$	N-n

that parallel windings may be considered to represent single phase transformer is constructed as in Fig. 2.9. The connection table 2.2 describes the admittances between different nodes for such a transformer with the assumption of off-nominal tappings only on the primary side.

c) Star-delta Transformer :

Using the same conventions and techniques as above, the lattice equivalent circuit of a three-phase star-delta transformer as a linear graph is shown in Fig. 2.10. The convention used for numbering the nodes and thus identifying

Table 2.2

Connection table for a delta-delta transformer with $\alpha = \sqrt{3}(1+t_{\alpha}), \beta = \sqrt{3}$ p.u.

Admittances	Between Nodes
y/α^2	A-B, B-C, C-A
$y/3$	a-b, b-c, c-a
$2y/\sqrt{3}\alpha$	A-a, B-b, C-c
$-y/\sqrt{3}\alpha$	A-b, B-c, C-a; a-B, b-C, c-A

opposite sides of the symmetrical lattice networks is as follows :

A-N/c-b; B-N/a-c; C-N/b-a

The connection Table 2.3 shows the admittances between various nodes. Also, for taps only on primary side, $t_{\beta} = 0$.

2.4 MODELLING OF THREE-PHASE TRANSMISSION LINES

An untransposed three phase transmission line is described by its phase impedance matrix for a three phase order as follows :

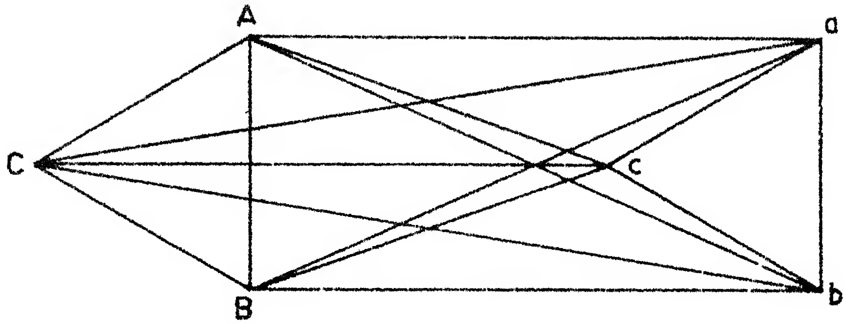


FIG. 2.9 SYMMETRICAL LATTICE EQUIVALENT CIRCUIT OF A 3-Ø DELTA-DELTA TRANSFORMER AS A LINEAR GRAPH.

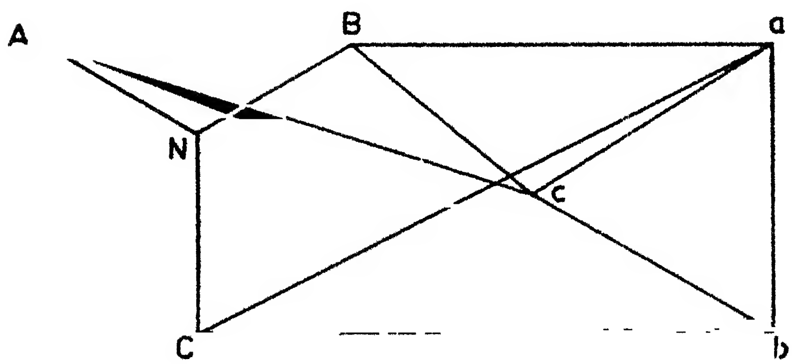


FIG. 2.10 SYMMETRICAL LATTICE EQUIVLENT CIRCUIT OF A 3-Ø STAR-DELTA TRANSFORMER AS A LINEAR GRAPH.

Table 2.3

Connection table for a star-delta transformer with

$\alpha = (1+t_\alpha)$, $\beta = \sqrt{3}$ p.u.

Admittances	Between Nodes
y/α^2	A-N, B-N, C-N
$y/3$	a-b, b-c, c-a
$y/\sqrt{3}\alpha$	A-c, B-a, C-b
$-y/\sqrt{3}\alpha$	A-b, B-c, C-a

$$Z_p^3 = \begin{bmatrix} Z_s & Z_{m1} & Z_{m2} \\ Z_{m2} & Z_s & Z_{m1} \\ Z_{m1} & Z_{m2} & Z_s \end{bmatrix} \qquad (2.35)$$

This coefficient matrix Z_p^3 can be diagonalized by the symmetrical component transformations.

Although, short lines can be represented by eqn. (2.35), the adequate representation of the medium and the long lines is obtained by nominal and equivalent π circuit representations. Fig. 2.11 shows schematically the series admittances Y_{abc} of a 3-phase transmission line between buses 1,2,3 and 4,5,6 and self and mutual admittances at any node 1.

Applying the KCL at node 1 , we get

$$(V_1-V_4)Y_{aa}+(V_1-V_5)(-Y_{ab})+(V_1-V_6)(-Y_{ac})+(V_1-V_2)Y_{ab}+(V_1-V_3)Y_{ac}=I_1 \tag{2.36}$$

or

$$[Y_{aa}\ -Y_{ab}\ -Y_{ac}\ -Y_{aa}\ Y_{ab}\ Y_{ac}] \begin{bmatrix} V_{123} \\ V_{456} \end{bmatrix} = [I_1] \tag{2.37}$$

Similarly, writing the KCL at all the nodes and combining them we get the following result

$$\begin{bmatrix} Y_{aa} & -Y_{ab} & -Y_{ac} & -Y_{aa} & Y_{ab} & Y_{ac} \\ -Y_{ba} & Y_{bb} & -Y_{bc} & Y_{ba} & -Y_{bb} & Y_{bc} \\ -Y_{ca} & -Y_{cb} & Y_{cc} & Y_{ca} & Y_{cb} & -Y_{cc} \\ -Y_{aa} & +Y_{ab} & +Y_{ac} & Y_{aa} & -Y_{ab} & -Y_{ac} \\ +Y_{ba} & -Y_{bb} & +Y_{bc} & -Y_{ba} & Y_{bb} & -Y_{bc} \\ +Y_{ca} & +Y_{cb} & -Y_{cc} & -Y_{ca} & -Y_{cb} & Y_{cc} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{2.38}$$

or,

$$\begin{bmatrix} Y_{abc} & -Y_{abc} \\ -Y_{abc} & Y_{abc} \end{bmatrix} \begin{bmatrix} V_{123} \\ V_{456} \end{bmatrix} = \begin{bmatrix} I_{123} \\ I_{456} \end{bmatrix} \tag{2.39}$$

where Y_{abc} is given by eqn. (2.8).

For equivalent π representation of the transmission line, we have also got to consider the shunt parameters at each end of the line as shown in Fig. 2.12. Hence, the final nodal voltages and currents relationships for the considered transmission lines is given by [7]

$$\begin{bmatrix} Y_{abc} + \frac{1}{2} Y_{shunt} & -Y_{abc} \\ -Y_{abc} & Y_{abc} + \frac{1}{2} Y_{shunt} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \qquad (2.40)$$

where

$$Y_{shunt} = \begin{bmatrix} y_{ao}+y_{ab}+y_{ac} & y_{ab} & y_{ac} \\ y_{ba} & y_{bo}+y_{bc}+y_{ab} & y_{bc} \\ y_{ca} & y_{cb} & y_{co}+y_{bc}+y_{ac} \end{bmatrix} \qquad (2.41)$$

If we have got two parallel lines, the couplings between the two can be considered as shown in Fig. 2.13. Each element in Fig. 2.13 represents a (3x3) admittance matrix. The bus admittance matrix for these two lines is given by

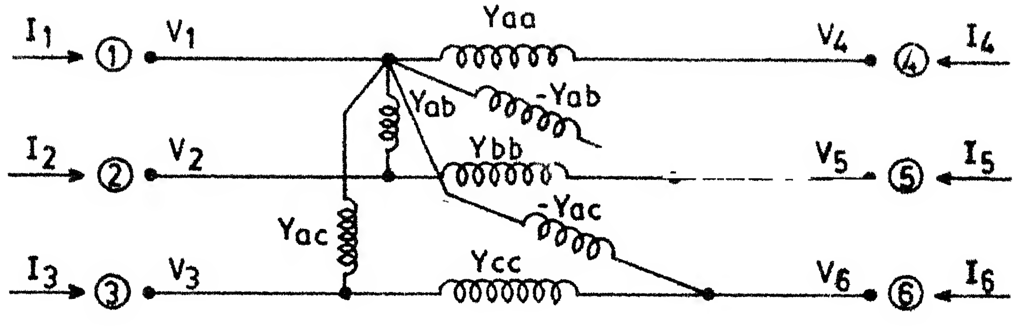


FIG. 2.11 SCHEMATIC REPRESENTATION OF SELF AND MUTUAL ADMITTANCES OF A 3-Ø TRANSMISSION LINE AT A NODE.

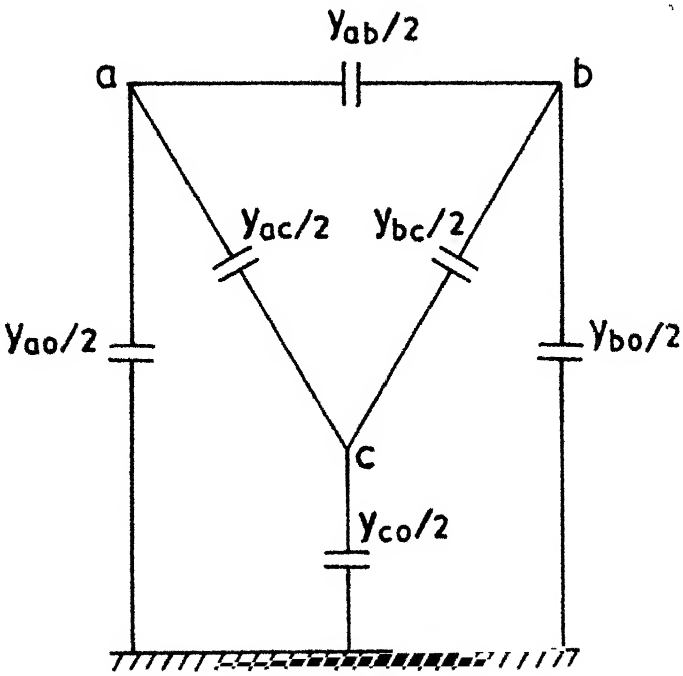


FIG. 2.12 THE CAPACITIVE SUSCEPTANCES AT ONE END OF A 3-Ø TRANSMISSION LINE.

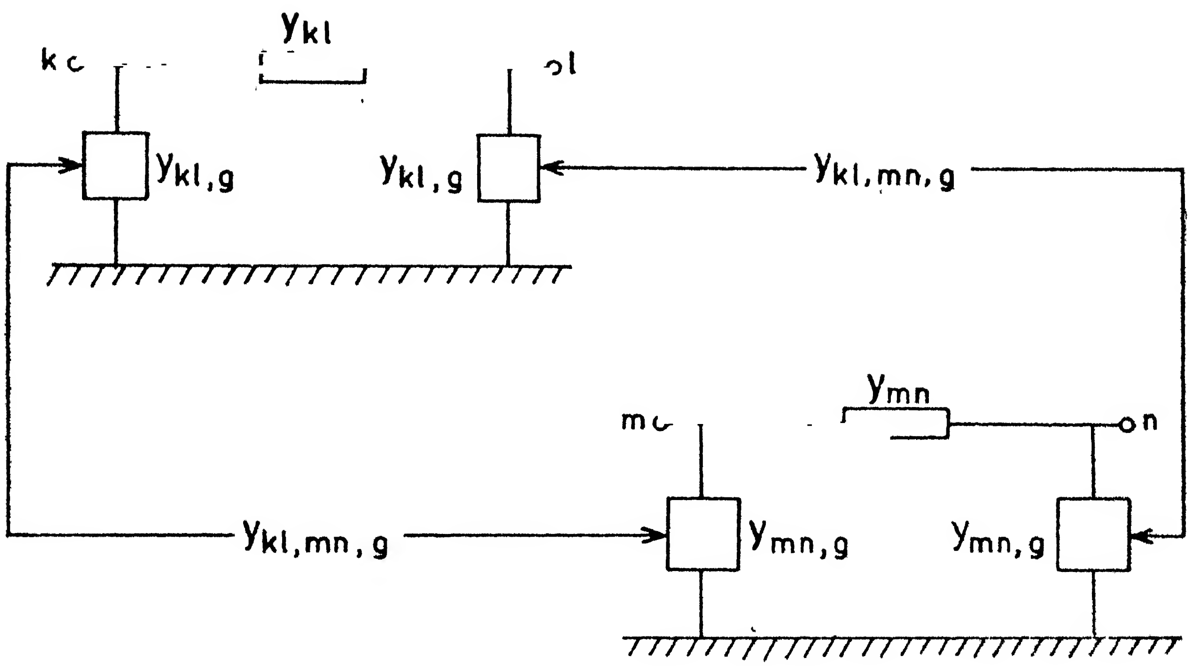


FIG. 2.13 THE MUTUAL COUPLING BETWEEN TWO PARALLEL THREE PHASE TRANSMISSION LINES.

$$\begin{array}{c|cccc|}
 & k & l & m & n \\
\hline
k & y_{kl}+y_{kl,g} & -y_{kl} & y_{kl,mn}+y_{kl,mn,g} & -y_{kl,mn} \\
l & -y_{kl} & y_{kl}+y_{kl,g} & -y_{kl,mn} & y_{kl,mn}+y_{kl,mn,g} \\
m & y_{kl,mn}+y_{kl,mn,g} & -y_{kl,mn} & y_{mn}+y_{mn,g} & -y_{mn} \\
n & -y_{kl,mn} & y_{kl,mn}+y_{kl,mn,g} & -y_{mn} & y_{mn}+y_{mn,g} \\
\hline
\end{array} \tag{2.42}$$

2.5 THREE PHASE LOADS

If three phase static loads are present in a power system, they are assumed to be decoupled and the admittance equivalent to one third of the load is added to the corresponding diagonal element of the system admittance matrix. It is calculated from the complex power and prefault nodal voltage. The load admittance is obtained from the following eqn. [20]

$$y_k = \frac{P_k - jQ_k}{V_k^*} \tag{2.43}$$

where k is any node.

Eqn. (2.43) is solved for each phase which becomes a bus in this case using the phase power and the phase nodal voltage. In the absence of any voltages, the three phase balanced voltage with unity magnitude may be used.

In case of a single phase load, its equivalent admittance is added to the corresponding diagonal element just like a shunt appearing between one of the nodes and the ground. The compensating reactors are treated like static load.

2.6 SAMPLE NETWORK

The single phase and three phase (balanced as well as unbalanced) load flow analysis of the sample power system [10,15] network as shown in Fig. 2.14 was carried out. The bus numbering sequences for single phase and three phase load flow have been shown separately in the figure. The necessary data for carrying out the analysis of the sample network are given in Table 2.4. The familiar Gauss-Seidel iterative technique was used in the study. The results obtained from the single phase load flow analysis with a tolerance of .00001 are shown in Tables 2.5 - 2.6. The computations were carried out on the DEC-1090 System at I.I.T., Kanpur, India.

Balanced Three Phase Load Flow Analysis :

The three phase balanced load flow analysis of the sample network shown in Fig. 2.14 was done in the phase frame of reference. The procedure for assembling the polyphase network nodal admittance matrix given by Roy [20] was employed. It may be noted, that, in three phase load flow studies, every phase at each busbar including the neutral of generator becomes a node. The main steps for carrying out the load flow analysis are :

i) Assign each phase of each busbar including neutral of the generator a node number.

ii) Form the system multiphase nodal admittance matrix using the representation of elements in the phase frame of reference.

iii) Form and then solve the nodal performance equation, $I = YV$, using some iterative technique (like Gauss-Seidel iterative technique in the present case)

iv) Compute the voltage magnitude, its angle at each bus and line flows, etc.

The results obtained from the balanced three phase load flow analysis, whose flow chart is given in Appendix , are given in Tables 2.7 - 2.8.

Unbalanced Load Flow Analysis :

The following unbalances were simulated in the sample network to evaluate the performance of the system.

i) The loads at node number 13,14,15 were unbalanced to 110 percent, 100 percent and 90 percent of their base case values.

ii) The delta winding of the transformer connected between the nodes 4,5,6 and 16,17,18 was converted to an open delta.

iii) The off-nominal tappings of the secondary side of the transformer connected between nodes 4,5,6 and 16,17,18 was set at -0.020 p.u.

The results obtained from the load flow analysis after making the changes as above are shown in Tables 2.7 - 2.8.

2.7 CONCLUSIONS

In this chapter, the necessity of representing the three-phase system in phase frame of reference has been emphasised. Consequently, the detailed mathematical models of the three phase generator, transformer, transmission line and load has been done. It has been shown that, more realistic performance of the system can be studied by doing phasor coordinate analysis of the system.

The following conclusions can be drawn, from the load flow analysis of the sample network done in the previous section :

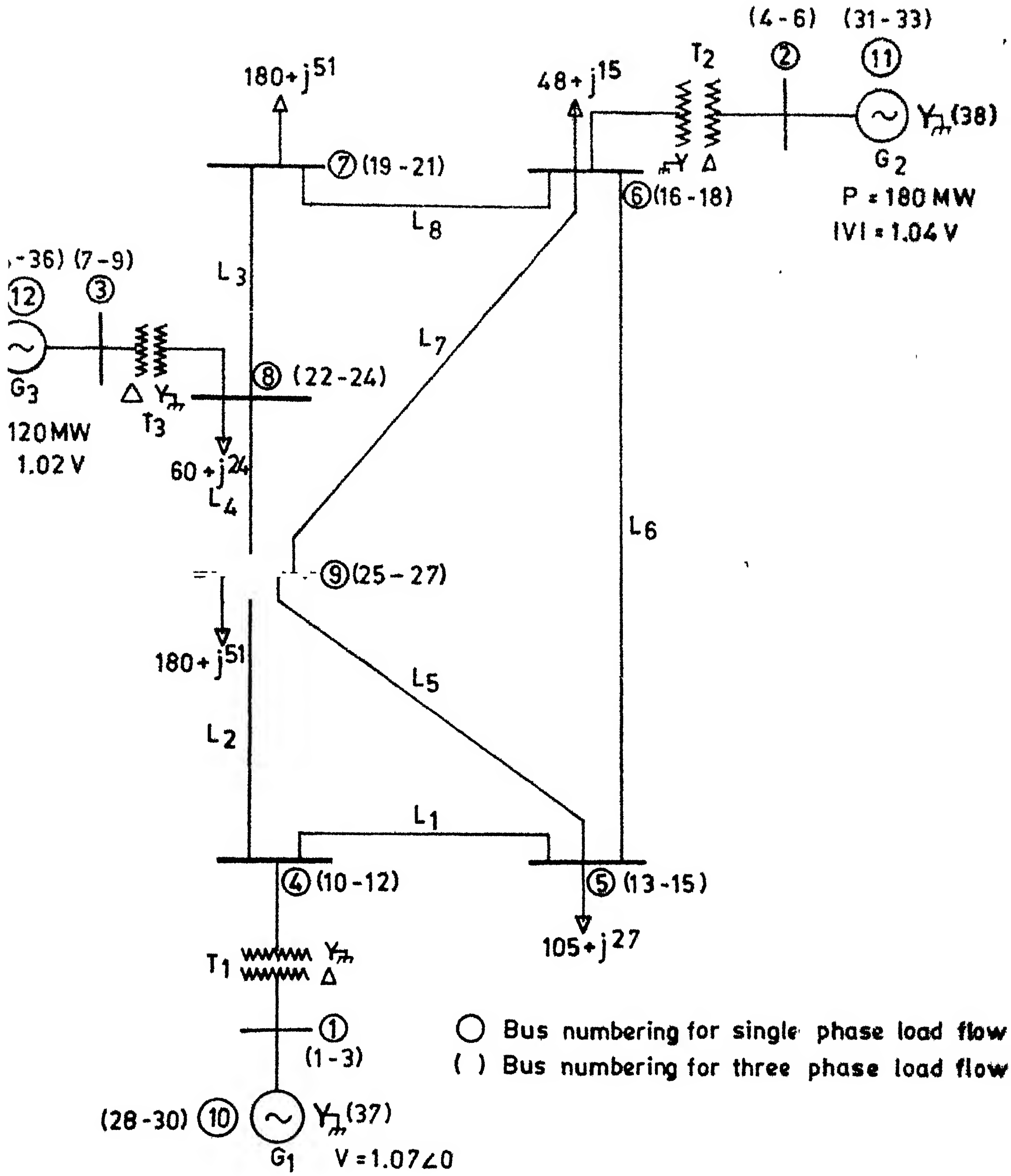
i) The results obtained from the single phase load flow analysis and the balanced three phase load flow analysis of the sample system are alike. This proves the validity of the single phase load flow analysis based on one line diagram when the system is balanced.

ii) When the system becomes unbalanced, single phase load flow fails to give accurate results as seen from the unbalanced load flow analysis.

iii) The unbalances in the magnitudes of voltages of different phases at a bus are considerable.

iv) It is seen, that, the line flows in the three phases between two buses are quite different from each other during unbalanced conditions.

The above results and the conclusions establish the need for conducting the load flow analysis of the system in phasor coordinates, especially, when there are unbalances in the system.



IG. 2.14 THE SAMPLE NETWORK WITH BUS NUMBERING SEQUENCE FOR SINGLE PHASE AND THREE PHASE LOAD FLOW STUDY.

Table 2.4

Circuit characteristics for sample system of Fig. 2.14 on 100 MVA base/phase and rated KV

From Bus	To Bus	± Sequence R	X	B	Zero sequence R_o	X_o	B_o
1) Generation							
(1-3)	(28-30)	0.0	0.0967	-	0.0	0.0467	-
(4-6)	(31-33)	0.0	0.17	-	0.0	0.0850	-
(7-9)	(34-36)	0.0	0.17	-	0.0	0.0850	-
2) Single circuit 3-Ø Lines :							
(25-27)	(22-24)	0.05	0.20	0.033	0.155	0.580	0.0193
(25-27)	(13-15)	0.10	0.40	0.066	0.310	1.160	0.0387
(16-18)	(13-15)	0.15	0.60	0.100	0.465	1.740	0.0580
(25-27)	(16-18)	0.10	0.40	0.066	0.310	1.160	0.0387
(19-21)	(16-18)	0.05	0.20	0.033	0.155	0.580	0.0193
3) Double circuit 3-Ø Lines :							
(10-12)	(13-15)	0.0375	0.150	0.00625	0.11625	0.435	0.00363
(10-12)	(25-27)	0.01875	0.075	0.00313	0.05813	0.2175	0.00180
(22-24)	(19-21)	0.025	0.100	0.00415	0.0775	0.2900	0.00240

4) Transformers :

From Bus	To Bus	Type of connection	Reactance X
(1-3)	(10-12)	Δ - Y_1	0.0533
(4-6)	(16-18)	Δ - Y_1	0.12
(7-9)	(22-24)	Δ - Y_1	0.16

Table 2.5

Voltage solution for single phase load flow of the network of Figure 2.14

Bus Number	Voltage Magnitude in p.u.	Angle in degree
1	1.043	-2.745
2	1.027	-2.642
3	1.005	-4.132
4	1.030	-1.465
5	1.023	-2.967
6	1.022	-3.746
7	0.989	-1.265
8	0.996	-4.943
9	1.018	-2.089
10	1.070	0.000
11	1.040	-1.378
12	1.020	-0.986

Table 2.6

Power Flows

From Bus	To Bus	Real Power (MW)	Reactive Power (MVAR)
10	1	276.554	98.746
11	2	179.295	34.193
12	3	139.836	30.574
8	9	-28.761	-30.792
5	9	- 0.498	-7.864
5	6	+14.231	+9.243
6	9	-22.902	-12.962
6	7	-93.807	- 24.024
5	4	-90.287	-9.356
9	4	-185.176	-52.262
7	8	+88.044	+22.575

Table 2.7

Voltage solution for three phase load flows of the network of Fig. 2.14

Bus No.	Balanced Case		Unbalanced Case	
	|V| (p.u.)	Θ (deg.)	|V| (p.u.)	Θ (deg.)
1	1.043	-4.616	1.040	-4.951
2	1.043	-124.686	1.034	-124.585
3	1.043	115.446	1.044	115.604
4	1.026	-2.831	1.029	89.329
5	1.026	-122.888	1.014	-29.109
6	1.026	117.189	1.040	-149.156
7	1.005	-5.617	1.002	-6.033
8	1.006	-125.675	0.996	-125.508
9	1.006	114.401	1.007	114.587
10	1.031	82.696	1.023	83.075
11	1.031	-37.231	1.034	-37.435
12	1.031	-157.281	1.018	-157.618
13	1.023	81.483	1.013	81.764
14	1.023	-38.444	1.029	-38.707
15	1.024	-158.489	1.007	-158.672
16	1.021	83.190	1.011	84.550
17	1.021	-36.740	1.031	-37.209
18	1.022	-155.768	0.981	-157.437
19	0.988	79.896	0.981	80.615
20	0.988	-40.035	0.994	-40.263
21	0.999	-160.060	0.963	-160.759
22	0.996	80.687	0.988	81.307
23	0.996	-39.242	1.000	-39.487
24	0.996	-159.270	0.976	-159.857
25	1.018	81.513	1.011	81.959
26	1.019	-38.415	1.023	-38.620
27	1.019	-158.458	1.002	-158.874
28	1.070	0.000	1.070	0.000
29	1.070	-120.072	1.070	-120.072
30	1.070	120.072	1.070	120.072
31	1.040	2.621	1.040	95.850
32	1.041	-117.436	1.041	-24.207
33	1.041	122.629	1.041	-144.215
34	1.020	-1.850	1.020	-1.862
35	1.021	-121.907	1.021	-121.919
36	1.021	118.158	1.021	118.146
37	0.000	0.000	1.021	0.000
38	0.000	0.000	0.000	0.000
39	0.000	0.000	0.000	0.000

Table 2.8

Power Flows

From Bus	Balanced Case		Unbalanced Case	
	Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)
28	92.919	33.974	99.341	38.021
29	92.901	33.691	90.083	43.307
30	93.129	33.860	90.026	32.727
31	59.656	11.698	70.727	10.923
32	59.761	11.889	53.109	18.730
33	59.565	11.802	55.140	-0.499
34	39.667	10.098	43.753	12.556
35	39.748	10.225	37.685	16.187
36	39.586	10.192	37.567	9.072
22	-9.507	-10.402	-8.175	-11.052
23	-9.557	-10.548	-10.042	-9.972
24	-9.412	-10.523	-10.526	-11.478
13	0.159	-2.285	-0.331	-2.374
14	0.162	-2.278	0.180	-2.311
15	0.154	-2.281	0.641	-2.087
13	-4.793	-3.654	-7.544	-2.209
14	-4.771	-3.597	-4.036	-4.757
15	-4.833	-3.610	-2.786	-0.128
16	7.367	-4.422	11.106	-6.198
17	7.337	-4.508	6.234	-2.962
18	7.423	-4.488	4.827	-9.080
16	31.465	8.041	36.144	5.818
17	31.452	8.031	30.231	10.061
18	31.470	8.031	27.867	1.061
13	-30.368	-3.061	-30.622	-5.317
14	-30.391	-3.122	-31.143	-1.935
15	-30.320	-3.109	-29.355	-5.885
25	-62.132	-17.570	-57.591	-20.641
26	-62.207	-17.786	-63.763	-15.477
27	-61.981	-17.742	-65.209	-23.952
19	-29.050	-7.707	-24.521	-10.560
20	-29.055	-7.711	-30.296	-5.50
21	-29.036	-7.712	-32.542	-14.454

CHAPTER 3

SIX PHASE POWER SYSTEM: MODELLING AND ANALYSIS

3.1 INTRODUCTION

Three phase power systems are the most widely used in the present day electric power systems. Although high phase order machine construction and power transmission have been under active consideration, specially during the last 10 years, still three phase systems continue to be in use. However, as the demand for power is increasing at a phenominal rate, multiphase systems are bound to become a reality in near future.

All the advantages of the multiphase systems stem from the fact that the phase angle between two phases is less resulting in lower adjacent phase voltages ~~angles~~ and also, lower line to line voltages between two adjacent lines.

Out of the higher phase orders proposed so far in papers and conferences (6,9 and 12), the six phase systems appear to be most promising. To carry out the planning and operation studies of six phase systems, their mathematical modelling should be known beforehand. Mathematical modelling is an easy and flexible means to understand the behaviour of the system.

This chapter deals mainly with the modelling aspect of various elements present in a six phase system. The chapter

starts with an overview of the feasibility of multiphase systems with special reference to six phase systems. The modelling starts with the development of an equation for a six phase machine including all the unbalances present in it. After that, representations of three/six-phase and six phase transformers have been proposed based upon their symmetrical lattice equivalent circuits. In the next section, the representation of a six-phase transmission line based upon its phase impedance matrix, π equivalent circuit and ABCD parameters has been given. To carry out the analysis of a composite three/six phase system, three phase equivalent representation of a six phase line and vice versa has been derived. This is followed by the modelling of six phase loads. Finally, the chapter concludes with the load flow analysis of a sample system along with the results and discussions.

3.2 FEASIBILITY OF MULTIPHASE SYSTEMS

The feasibility studies of multiphase systems, in particular, six phase systems, are available in the literature [2-6,8-11] with regard to their steady-state operation, overvoltages, insulation requirements, power flow, reliability, etc. It has been found [2-3] that in several features, six phase systems possess better characteristics than that of the three phase systems. The summary of the relative merits and demerits [2-3] of the multiphase systems is given below.

(a) Power Transfer Capability :

More power can be transferred with multiphase systems and smaller line and tower dimensions are required. It has been found out that, if an existing double circuit three phase line is converted to a six phase line with the same adjacent phase-phase voltages in both the cases, the power transfer capability gets increased by 73.2 percent.

(b) Conductor Clearance :

If the system voltage is defined with phase-ground voltage as references, we find that adjacent phase-phase voltage decreases with phase order. Hence, line conductor spacing reduces with increased phase order. But there is a limit to reduction of spacings because of motion of individual conductors owing to ice, wind, faults, etc.

(c) Transposition :

It is difficult to transpose multiphase lines unlike the three phase lines which can be freely transposed. The only practical transposition for multiphase lines is obtained by rotating the entire conductor array in steps over the length of the line in a cyclic manner.

(d) Thermal Loading :

If the thermal loading is the criterion for circuit rating, the capacity increase is proportional to the number

of phases as the thermal loading is directly proportional to the number of phases.

(e) Surge Impedance Loading :

The Surge Impedance Loading of a transmission line is almost proportional to the phase order increase and reaches saturation beyond six-phase.

(f) Current Unbalance :

Current unbalances in multiphase lines are less than that of three phase lines. Hence, six phase circuits are better balanced compared to two three phase circuits with the same conductor configurations. Hence, transposition may be unnecessary in the former case.

(g) Electric Fields :

Shield wires may be required for high phase orders to compensate the effect of decrease in maximum surface electric field and increase in maximum ground electric field.

(h) Radio and Audible Noise :

The performance of multiphase systems is better than their three phase counterparts as far as radio and audible noise is concerned.

(i) Fault Overvoltages :

For a six phase system, fault overvoltages are slightly higher than those in a comparable three phase system.

(j) Switching Surges :

Although phase to ground switching surges are approximately same for three and six phase systems under similar conditions, phase to phase surges increase with phase order. This may impose an upper limit to achievable phase order.

(k) Rate of Rise of Recovery Voltage

For the same short circuit MVA, Rate of rise of recovery voltage across the breaker terminals during normal opening is less in higher phase order systems.

(l) Lightning Performance :

Due to the reduced dimensions of six phase lines, the lightning strokes to the line are about 20 percent less than those in three phase systems. However, the probability of phase-phase flashover is much higher on a six-phase line. Thus, assuming effective shielding, the overall lightning performance of high phase order lines is comparable to conventional three phase line.

(m) Terminal Insulation Level :

The terminal insulation level is slightly higher for six phase systems than for three phase systems or for systems of 12 phase and higher.

(n) Reliability :

The reliability study of a six phase line [5] obtained

from a double circuit three-phase line based upon load flow analysis, stability analysis and stochastic reliability reveals that,

(i) Voltage regulation and efficiency are better with six phase lines

(ii) A six phase line seems to be more stable than a double circuit three-phase line

(iii) More energy demand can be met at a load point by converting a double circuit three phase line to a six phase line

(iv) The loss of a six-phase li[illegible]ay have greater consequences on power system than the loss of a double circuit three phase line.

Although, multi-phase lines are not being used in practice, developments and tests in this field have shown that a multiphase transmission system is realizable and may find applications in narrower rights of way, uprating and multi-circuit configurations in entire transmission range of voltages.

3.3 MATHEMATICAL MODELLING OF SIX PHASE MACHINES

Although, the topic of feasibility of a six phase machine is still in its infancy, it has been proposed to develop six phase machines for high power applications.

In this section, a six phase synchronous generator has been modelled with a grounded reactance Y_{NO} in the same manner as the three phase generator . The general six phase element can be represented as shown in Fig. 3.1. The equation of this element in the impedance form can be written as

$$V^6_{p'} + E^6_p = Z^6_p I^6_{p'} + V^6_p \tag{3.1}$$

where

$$V^6_{p'} = [V_7\ V_8\ V_9\ V_{10}\ V_{11}\ V_{12}]^T ,$$

$$E^6_p = [E_a\ E_b\ E_c\ E_d\ E_e\ E_f]^T ,$$

$$V^6_p = [V_1\ V_2\ V_3\ V_4\ V_5\ V_6]^T ,$$

$$Z^6_p = 6\text{x}6 \text{ impedance matrix} = [Y^6_p]^{-1} ,$$

and $$I^6_{p'} = [I_7\ I_8\ I_9\ I_{10}\ I_{11}\ I_{12}]^T$$

Thus, the nodal equations for the voltage and current directions shown in Fig. 3.1 can either be written from the eqn. (3.1) or derived from the symmetrical lattice equivalent circuit in the similar manner to three phase as follows :

$$\begin{bmatrix} I^6_p \\ I^6_{p'} \end{bmatrix} = \begin{bmatrix} Y^6_p & -Y^6_p & -Y^6_p \\ -Y^6_p & Y^6_p & Y^6_p \end{bmatrix} \begin{bmatrix} V^6_p \\ V^6_{p'} \\ E^6_p \end{bmatrix} \tag{3.2}$$

where, $I^6_p = [I_1\ I_2\ I_3\ I_4\ I_5\ I_6]^T$

Fig. 3.1 can represent a six phase machine if E_a, E_b, E_c, E_d, E_e and E_f are a balanced set of internal voltages and nodes 7-12 are joined to form a star point. Hence, a six phase generator is represented by a balanced set of internal voltages behind proper reactances. For a balanced set of internal voltages,

$$E_b = \alpha^* E_a,\ E_c = -\alpha E_a,\ E_d = -E_a,\ E_e = -\alpha^* E_a,\ E_f = \alpha E_a$$

i.e.,

$$E_p^6 = [1 \quad \alpha^* \quad -\alpha \quad -1 \quad -\alpha^* \quad \alpha]^T E_a \tag{3.3}$$

where, $\alpha = e^{j2\pi/6} = 0.50 + j0.866 = 6\sqrt{1}$

If the nodes 7-12 are joined to form a star point, then

$$I_N = I_7+I_8+I_9+I_{10}+I_{11}+I_{12} \tag{3.4}$$

and

$$V_N = V_7 = V_8 = V_9 = V_{10} = V_{11} = V_{12} \tag{3.5}$$

Hence, internal voltages become

$$E_p^6 = [E_a+V_N\,,\ E_b+V_N\,,\ \ldots\quad E_f+V_N]^T \tag{3.6}$$

and the current vector I_p^6, is given by

$$I_a = \frac{S^*}{(V_N+E_a)^*}\ ;\quad I_b = \frac{S_b^*}{(V_N+E_b)^*}\ ;\ \ldots\ ;\ I_f = \frac{S_f^*}{(V_N+E_f)^*} \tag{3.7}$$

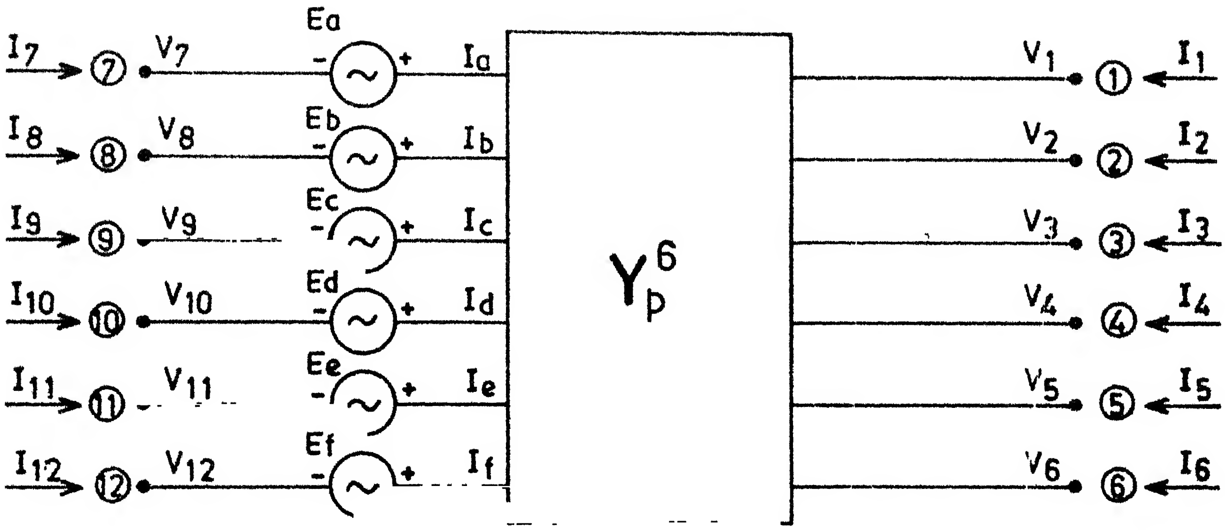


FIG. 3.1 SCHEMATIC REPRESENTATION OF A 6-Ø ELEMENT

$$Y_p^6 = \begin{bmatrix} Y_s & Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} \\ Y_{m5} & Y_s & Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} \\ Y_{m4} & Y_{m5} & Y_s & Y_{m1} & Y_{m2} & Y_{m3} \\ Y_{m3} & Y_{m4} & Y_{m5} & Y_s & Y_{m1} & Y_{m2} \\ Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} & Y_s & Y_{m1} \\ Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} & Y_{m5} & Y_s \end{bmatrix} \tag{3.8}$$

Here Y_p^6 is obtained by

$$Y_p^6 = [T_s]\ [Y_{comp}^6][T_s^*]^T \tag{3.9}$$

where T_s is a six phase symmetrical component transformation matrix [21,22] given by

$$T_s^6 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha^* & -\alpha & -1 & -\alpha^* & \alpha \\ 1 & -\alpha & -\alpha^* & 1 & -\alpha & -\alpha^* \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\alpha^* & -\alpha & 1 & -\alpha^* & -\alpha \\ 1 & \alpha & -\alpha^* & -1 & \alpha & \alpha^* \end{bmatrix} \tag{3.10}$$

and $Y_{comp}^6 = \text{diag.}\ [y_o, y_1, y_2, y_3, y_4, y_5]$ (3.11)

After transformation, various terms in the matrix Y_p^6 (eqn. 3.8) are shown

$$Y_s = \frac{1}{6}\,(y_o + y_1 + y_2 + y_3 + y_4 + y_5)$$

$$Y_{m1} = \frac{1}{6}(y_o + \alpha y_1 - \alpha^* y_2 - y_3 - \alpha y_4 + \alpha^* y_5)$$

$$Y_{m2} = \frac{1}{6}(y_o - \alpha^* y_1 - \alpha y_2 + y_3 - \alpha^* y_4 - \alpha y_5)$$

$$Y_{m3} = \frac{1}{6}(y_o - y_1 + y_2 - y_3 + y_4 - y_5)$$

$$Y_{m4} = \frac{1}{6}(y_o - \alpha y_1 - \alpha^* y_2 + y_3 - \alpha y_4 - \alpha^* y_5)$$

$$Y_{m5} = \frac{1}{6}(y_o + \alpha^* y_1 - \alpha y_2 - y_3 - \alpha^* y_4 + \alpha y_5)$$

Now substituting the values of Y_p^6, E_p^6 and $I_p'^6$ from eqns. (3.8), (3.6) and (3.7) respectively in eqn. (3.2), we get upon simplification :

$$\left[\begin{array}{c|cc}
 & -y_o & -y_1 \\
 & -y_o & -\alpha^* y_1 \\
Y_p^6 & -y_o & \alpha y_1 \\
 & -y_o & y_1 \\
 & -y_o & \alpha^* y_1 \\
 & -y_o & -\alpha y_1 \\
\hline
 & +y_o & y_1 \\
 & +y_o & \alpha^* y_1 \\
 & +y_o & -\alpha y_1 \\
-Y_p^6 & +y_o & -y_1 \\
 & +y_o & -\alpha^* y_1 \\
 & +y_o & \alpha y_1
\end{array}\right]
\left[\begin{array}{c}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\
\hline
 \\ V_N \\ \\ E_a \\ \\ \\
\end{array}\right]
=
\left[\begin{array}{c}
I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\
\hline
S_a^*/(V_N+E_a)^* \\
S_b^*/(V_N+E_b)^* \\
S_c^*/(V_N+E_c)^* \\
S_d^*/(V_N+E_d)^* \\
S_e^*/(V_N+E_e)^* \\
S_f^*/(V_N+E_f)^*
\end{array}\right]
\qquad (3.12)$$

For Node N ,an additional eqn. may be written by adding last six eqn. (3.12) as seen from eqn. (3.4) as follows :

$$[-y_o \quad -y_o \quad -y_o \quad -y_o \quad -y_o \quad -y_o \quad 6y_o \quad 0] \begin{bmatrix} V_p^6 \\ V_N \\ E_a \end{bmatrix} = [I_N] \quad (3.13)$$

From eqn. (3.3) we can deduce that

$$E_b^* = \alpha E_a^*, \; E_c^* = -\alpha^* E_a^*, \; E_d^* = -E_a^*, \; E_e^* = -\alpha E_a^*, \; E_f^* = \alpha^* E_a^*$$

Cross multiplying the last six eqns. of (3.12), adding them and simplifying the expression gives the following result :

$$-y_oV_1V_N^* - y_oV_2V_N^* - y_oV_3V_N^* - y_oV_4V_N^* - y_oV_5V_N^* - y_oV_6V_N^*$$

$$+6y_oV_NV_N^* - y_1V_1E_a^* - \alpha y_1V_2E_a^* + \alpha^* y_1V_3E_a^* + y_1V_4E_a^*$$

$$+\alpha y_1V_5E_a^* - \alpha^* y_1V_6E_a^* + 6y_1E_aE_a^* = S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^* \quad (3.14)$$

We note from the first seven terms of the eqn. (3.14) that they are V_N^* times eqn. (3.13). If no current is injected through the neutral which is the usual condition, $I_N = 0$ and eqn. (3.14) simplifies to

$$[-y_1 \quad -\alpha y_1 \quad \alpha^* y_1 \quad y_1 \quad \alpha y_1 \quad -\alpha^* y_1 \quad 6y_1] \begin{bmatrix} V_p^6 \\ \\ E_a \end{bmatrix} = \frac{S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^*}{E_a^*} \quad (3.15)$$

If the neutral is earthed through an admittance y_{NO}, eqn. (3.4) is changed to

$$I_N = I_7+I_8+I_9+I_{10}+I_{11}+I_{12}+y_{NO}\, V_N \tag{3.16}$$

Thus from eqn. (3.13), we can write

$$[-y_o \ -y_o \ -y_o \ -y_o \ -y_o \ -y_o \ 6y_o+y_{NO}] \begin{bmatrix} V_p^6 \\ \\ V_N \end{bmatrix} = [I_N] \tag{3.17}$$

Substituting the value of $V_N^* I_N$ in eqn. (3.14), we get with $I_N = 0$

$$[-y_1 \ -\alpha y_1 \ \alpha^* y_1 \ y_1 \alpha y_1 \ -\alpha^* y_1 \ 6y_1] \begin{bmatrix} V_p^6 \\ \\ E_a \end{bmatrix} = \frac{S_a^*+S_b^*+S_c^*+S_d^*+S_e^*+S_f^*}{E_a^*} + \frac{y_{NO}|V_N|^2}{E_a^*} \tag{3.18}$$

Thus the final machine equation (with $I_N = 0$) of the six phase machine are the first six equations of (3.12), eqn. (3.17) and eqns. (3.18) as shown,

$$
\left[\begin{array}{cccccc|c|c}
 & & & & & & -y_1 & -y_o \\
 & & & & & & -\alpha^* y_1 & -y_o \\
 & & Y_p^6 & & & & \alpha y_1 & -y_o \\
 & & & & & & y_1 & -y_o \\
 & & & & & & \alpha^* y_1 & -y_o \\
 & & & & & & -\alpha Y_1 & -Y_o \\
\hline
-y_1 & -\alpha y_1 & \alpha^* y_1 & y_1 & \alpha y_1 & -\alpha^* y_1 & 6y_1 & 0 \\
\hline
-y_o & -y_o & -y_o & -y_o & -y_o & -y_o & 0 & 6y_o + y_{NO}
\end{array}\right]
\left[\begin{array}{c} V_p^6 \\ \hline E_a \\ \hline V_N \end{array}\right]
=
\left[\begin{array}{c} I_p^6 \\ \hline S^*/E_a^* \\ \hline 0 \end{array}\right]
\qquad (3.19)
$$

In the eqn. (3.19), the term $\frac{y_{NO}|V_N|^2}{E_a^*}$ has been neglected as it is negligible as compared to the total power in eqn. (3.18). Eqn. (3.19) can also be written as

$$
\begin{bmatrix}
Y_p^6 & -\bar{Y}_1 & -\bar{Y}_o \\
-\bar{Y}_2 & 6y_1 & 0 \\
-\bar{Y}_o^T & 0 & (6y_o + y_{NO})
\end{bmatrix}
\begin{bmatrix} V_p^6 \\ E_a \\ V_N \end{bmatrix}
=
\begin{bmatrix} I_p^6 \\ S^*/E_a^* \\ 0 \end{bmatrix}
\qquad (3.20)
$$

where, $S^* = S_a^* + S_b^* + S_c^* + S_d^* + S_e^* + S_f^*$,

$$\bar{Y}_1 = [1 \ \alpha^* \ -\alpha \ -1 \ -\alpha^* \ \alpha]^T \ y_1$$

$$\bar{Y}_2 = [1 \ \alpha \ -\alpha^* \ -1 \ -\alpha \ \alpha^*]^T \ y_1$$

$$\bar{Y}_o = [1 \ 1 \ 1 \ 1 \ 1 \ 1]^T \ y_o$$

Eqns. (3.19) or (3.20) is the final eqn. of a six phase machine capable of incorporating all the possible unbalances in it. This equation is used to form the admittance matrix of six phase synchronous generators.

3.4 MODELLING OF TRANSFORMERS

The multiphase transformers are normally used to obtain 6,9,12 and higher phase conversions from three phase systems. A six-phase conversion can be obtained from a three-phase system by using wye/star, delta/star or some other connections. For getting still higher phase conversion, specially constructed units are used. In this section, symmetrical lattice equivalent circuits of two possible transformer connections mentioned above and six-phase transformers have been employed to obtain the admittance submatrix. The models of other schemes may be developed in a similar manner.

3.4.1 Three Phase/Six Phase Transformers

The symmetrical lattice equivalent circuit of single phase units given in Fig. 2.7 is employed to obtain the model of three/six phase transformers. Since most of the time, we have got tappings on only one side of the transformer, they have been assumed to be t_α on the primary side and those on the secondary side are taken as zero. The parallel windings of three-phase/six phase transformer are assumed to represent an equivalent single phase transformer.

Consider a wye/star three phase/six phase transformer shown in Fig. 3.2. It can be conceptualized to be made of three windings [8,9,10] as follows :

a) a three-phase winding P with terminal a,b,c and neutral N

b) a secondary equivalent three phase winding S having terminals 1,5,3 and neutral n

c) a tertiary equivalent three-phase winding T with terminals 4,6,2 and neutral n having 180° phase shift with respect to the winding S with the neutrals of windings S and T joined together to form a common neutral n.

An equivalent circuit of this transformer can be assembled by considering three wye/wye equivalent circuits in parallel as shown in Fig. 3.3. If y_{PS}, y_{PT}, y_{ST} are the parameters of the transformer, where y_{PS} in the leakage impedance referred to the primary side with secondary short circuited and tertiary open circuited, and so on, the connection table of the transformer can be obtained as shown in Table 3.1. The three windings P,S and T have turns ratios of αT_1 1 and -1 respectively. Since winding T is exactly 180° opposite in phase to winding S, its turn ratio is negative to that of S. Table 3.1 has been derived in a similar manner as done earlier in the case of three phase transformers.

Table 3.1

Connection table for wye/star three phase/six phase transformer with $\alpha_T = (1+t_\alpha), \beta_T = 1, \gamma_T = -1$ p.u.

Admittance	Between Nodes
$(y_{PS}+y_{PT})/\alpha_T^2$	N-a, N-b, N-c
$y_{PS}+2y_{ST}$	n-1, n-3, n-5
$y_{PT}+2y_{ST}$	n-4, n-6, n-2
$(y_{PT}-y_{PS})/\alpha_T$	n-a, n-b, n-c
y_{PS}/α_T	a-1, b-3, c-5
$-y_{PS}/\alpha_T$	N-1, N-3, N-5
$-y_{PT}/\alpha_T$	a-4, b-6, c-2
y_{PT}/α_T	N-4, N-6, N-2
$-y_{ST}$	1-4, 3-6, 5-2

From Table 3.1, the nodal admittance matrix of the above transformer can be obtained as shown :

$$
\begin{array}{c|ccc|cccccc|}
 & a & b & c & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
a & y_1 & & & -y_4 & & & y_5 & & \\
b & & y_1 & & & & -y_4 & & & y_5 \\
c & & & y_1 & & y_5 & & & -y_4 & \\
\hline
1 & -y_4 & & & y_2 & & & y_6 & & \\
2 & & & y_5 & & y_3 & & & y_6 & \\
3 & & -y_4 & & & & y_2 & & & y_6 \\
4 & y_5 & & & y_6 & & & y_3 & & \\
5 & & & -y_4 & & y_6 & & & y_2 & \\
6 & & y_5 & & & & y_6 & & & y_3 \\
\hline
\end{array}
\qquad (3.21)
$$

where, $y_1 = (y_{PS}+y_{PT})/\alpha_T^2$; $y_2 = (y_{PS}+y_{PT})$

$y_3 = (y_{PT}+y_{ST})$; $y_4 = y_{PS}/\alpha_T$

$y_5 = y_{PT}/\alpha_T$; $y_6 = y_{ST}$

The nodal admittance matrix of a delta/star transformer shown in Fig. 3.4 can also be obtained in a similar-manner. In this case, also, we can visualize the transformer as consisting of three windings with the change from the case of wye/star transformer that primary is a delta connected three phase winding with turns ratio $\alpha_T = \sqrt{3}\ (1+t_\alpha)$, where t_α is the off-nominal tapping on the primary side. Thus, the equivalent circuit

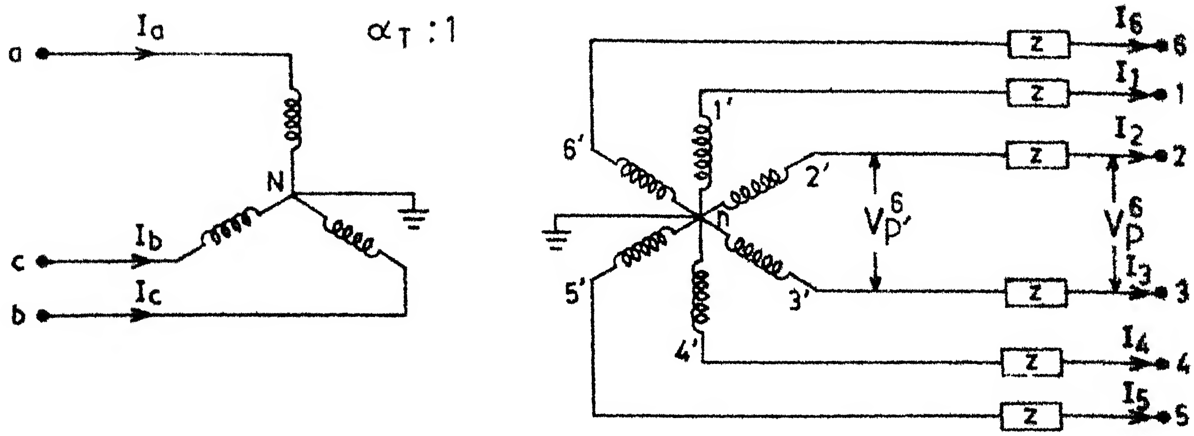


FIG. 3.2 A THRE PHASE/SIX PHASE, WYE/STAR TRANSFORMER OF TURNS RATIO $\alpha_T : 1$

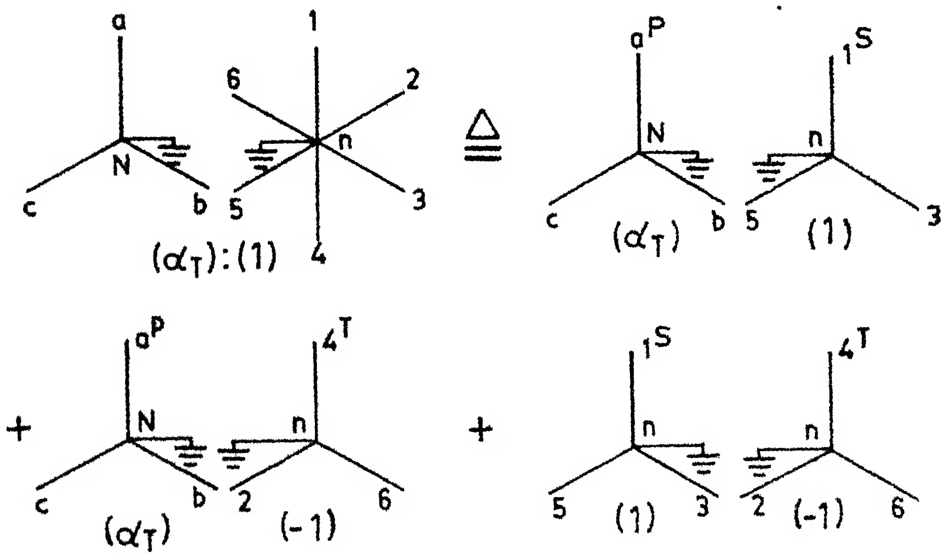


FIG. 3.3 REPRESENTATION OF A WYE/STAR TRANSFORMER FOR OBTAINING AN EQUIVALENT CIRCUIT.

of three phase/six phase delta/star transformer is obtained by paralleling two delta/wye and one wye/wye equivalent circuits in turn as shown in Fig. 3.5. Table 3.2 gives the connections showing admittances between various nodes with the same assumption that parallel windings represent single phase units.

Table 3.2

Connection table for delta/star three/six phase transformer with $\alpha_T = \sqrt{3}(1+t_\alpha)$, $\beta_T=1$, $\gamma_T = -1$ p.u.

Admittance	Between Nodes
$(y_{PS}+y_{PT})/\alpha_T^2$	a-b, b-c, c-a
$(y_{PS}+2y_{ST})$	1-n, 3-n, 5-n
$(y_{PT}+2y_{ST})$	4-n, 6-n, 2-n
y_{PS}/α_T	1-c, 3-a, 5-b
$-y_{PS}/\alpha_T$	1-b, 3-c, 5-a
$-y_{PT}/\alpha_T$	4-c, 6-a, 2-b
y_{PT}/α_T	4-b, 6-c, 2-a
$-y_{ST}$	1-4, 3-6, 5-2

The nodal admittance matrix obtained from Table 3.2 is given below :

$$
Y_{TR2} = \begin{array}{c|ccc|cccccc|}
 & a & b & c & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
a & y_1 & y_2 & y_2 & & y_3 & y_4 & & -y_4 & -y_3 \\
b & y_2 & y_1 & y_2 & -y_4 & -y_3 & & y_4 & y_3 & \\
c & y_2 & y_2 & y_1 & y_4 & & -y_4 & -y_3 & & y_3 \\
\hline
1 & & -y_4 & y_4 & & & & -y_5 & & \\
2 & y_3 & -y_3 & & & & & & -y_5 & \\
3 & y_4 & & -y_4 & & & & & & -y_5 \\
4 & & y_4 & -y_3 & -y_5 & & & & & \\
5 & -y_4 & y_3 & & & -y_5 & & & & \\
6 & -y_3 & & y_3 & & & -y_5 & & & \\
\hline
\end{array} \qquad (3.22)
$$

where, $y_1 = 2(y_{PS}+y_{PT})/\alpha_T^2$, $\quad y_2 = (y_{PS}+y_{PT})/\alpha_T^2$,

$y_3 = y_{PT}/\alpha_T$, $\quad y_4 = y_{PS}/\alpha_T$, $\quad y_5 = y_{ST}$

3.4.2 Six Phase Transformers

Consider a star/star six phase transformer shown in Fig. 3.6. Assuming that parallel windings represent single units,

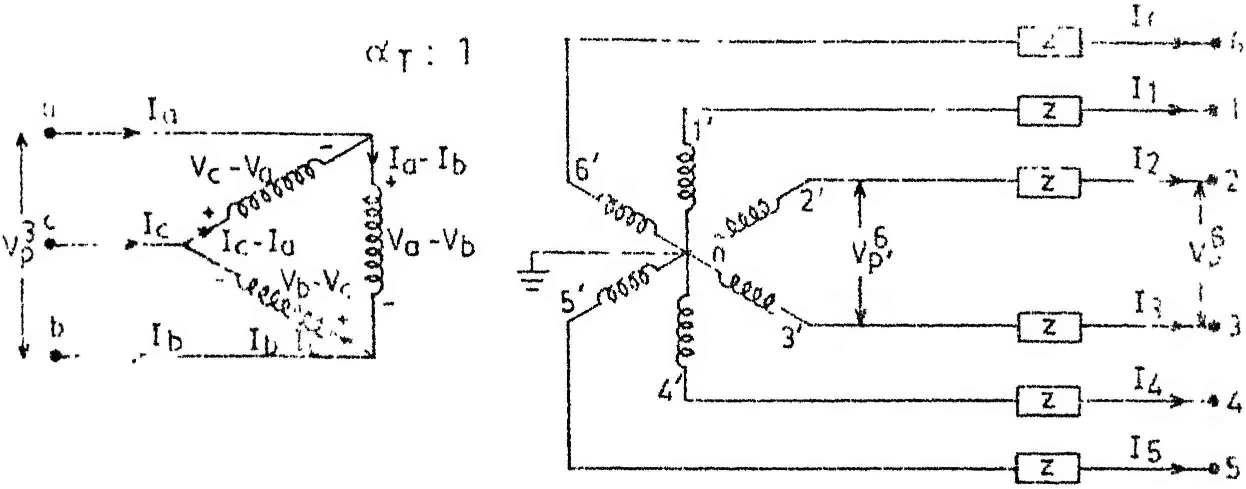


FIG. 3.4 A THREE-PHASE/SIX PHASE DELTA/STAR TRANSFORMER OF TURNS RATIO $\alpha_T:1$

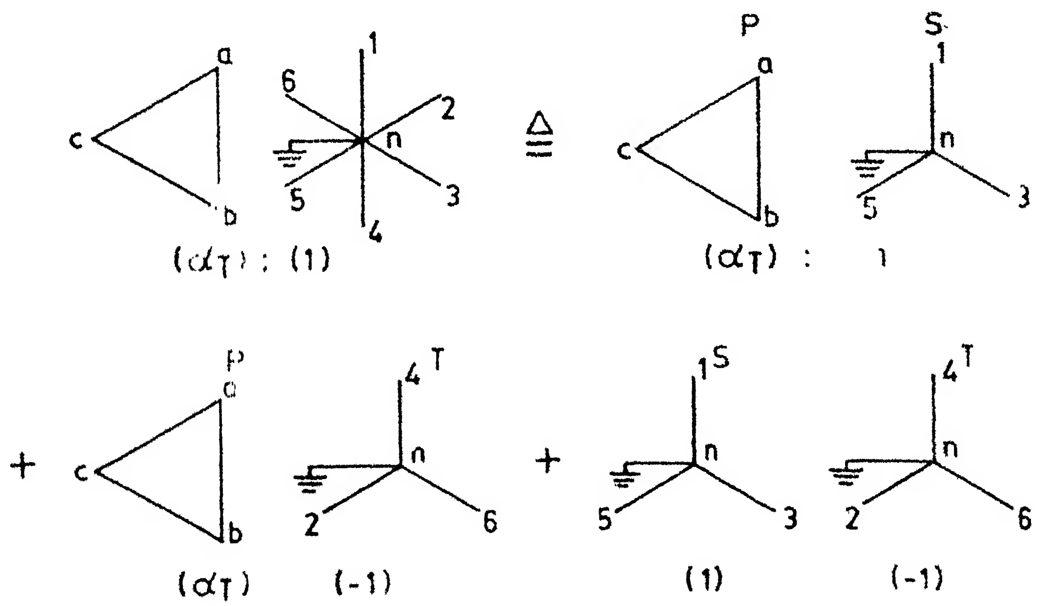


FIG. 3.5 REPRESENTATION OF A DELTA/STAR TRANSFORMER FOR OBTAINING AN EQUIVALENT CIRCUIT.

admittances between various nodes are obtained as shown in Table 3.3.

Table 3.3

Connection table for a six phase star/star transformer with $\alpha_T = (1+t_\alpha)$, $\beta_T = 1$ p.u.

Admittance	Between Nodes
y/α_T^2	N-a, N-b, N-c, N-d, N-e, N-f
y	n-1, n-2, n-3, n-4, n-5, n-6
y/α_T	a-1, b-2, c-3, d-4, e-5, f-6
$-y/\alpha_T$	n-a, n-b, n-c, n-d, n-e, n-f
$-y/\alpha_T$	N-1, N-2, N-3, N-4, N-5, N-6
$6y/\alpha_T$	N-n

The nodal admittance matrix for such transformer as obtained from Table 3.3 is

$$Y_{TR3} = \begin{array}{c|cc} & a \ldots f & 1 \ldots 6 \\ \hline a \ldots f & Y_{T1} & Y_{T2} \\ 1 \ldots 6 & Y_{T2} & Y_{T3} \end{array} \qquad (3.23)$$

where,

$$Y_{T1} = \text{diag. } [1\ 1\ 1\ 1\ 1\ 1](y/\alpha_T^2)$$

$$Y_{T2} = \text{diag. } [1\ 1\ 1\ 1\ 1\ 1](-y/\alpha_T)$$

$$Y_{T3} = \text{diag. } [1\ 1\ 1\ 1\ 1\ 1]y$$

where y is the series leakage admittance in p.u. of the transformer.

In deriving the representations for transformers, the effect of magnetising core and saturation are neglected. It has been found out that the models derived above are quite precise for unbalanced studies of the system. The representations for other types of transformer can also be obtained in the manner illustrated above.

3.5 MODELLING OF TRANSMISSION LINES

It is necessary to derive the models of transmission line which forms an integral part of the power systems. In this section, the representation of multiphase lines in terms of phase impedance matrix, π-circuit representation and ABCD parameters have been developed. Three phase equivalent representation of a six-phase line and vice versa have also been done.

a) Phase Impedance Matrix :

A six phase short untransposed transmission line processing cyclic symmetry is described by its phase impedance matrix as shown

$$Z_p^6 = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} \end{bmatrix} \tag{3.24}$$

This matrix can be fully diagonalized by using symmetrical component six phase transformations. However, medium and long lines cannot be representation by (3.24).

b) π-Circuit Representation :

The three phase nominal and equivalent π-circuit representation for three phase systems can be extended to six phase systems as well to adequately represent medium and long lines as shown :

$$\begin{bmatrix} I_{pS}^6 \\ I_{pR}^6 \end{bmatrix} = \begin{bmatrix} Y_p^6 + \frac{1}{2} Y_{sh}^6 & -Y_p^6 \\ -Y_p^6 & Y_p^6 + \frac{1}{2} Y_{sh}^6 \end{bmatrix} \begin{bmatrix} V_{pS}^6 \\ V_{pR}^6 \end{bmatrix} \tag{3.25}$$

The derivation of eqn. (3.25) can be done on the same lines as for three phase transmission lines. In eqn. (3.25), the subscripts S and R denote sending and receiving ends respectively, Y_p^6 is a (6x6) phase admittance matrix obtained from the relation (3.9).

c) ABCD-Parameters Representation :

The representation of six phase transmission lines using ABCD parameters can be written as follows :

$$\begin{bmatrix} V_{pS}^6 \\ I_{pS}^6 \end{bmatrix} = \begin{bmatrix} A^6 & B^6 \\ C^6 & D^6 \end{bmatrix} \begin{bmatrix} V_{pR}^6 \\ I_{pR}^6 \end{bmatrix} \tag{3.26}$$

d) Equivalent Three Phase Representation of a Six-Phase Line :

If we are interested in carrying out the analysis of a composite three phase six phase system, the interest of investigation being on the three phase part of it, we have to formulate an equivalent three phase representation of the six phase part of the system. For example, consider a system having a six phase transmission line connected between three phase buses S and R via three phase/six phase transformers TR_1 and TR_2 as shown in Fig. 3.7. The equation of the transmission line connected between buses S' and R' in impedance form is written as

$$V_{S'}^6 - V_{R'}^6 = Z_p^6 I_p^6 \tag{3.27}$$

Assuming the transformers TR_1 and TR_2 to be wye/star, three phase/six phase transformers, we can write the expressions for three phase voltages V_S^3 and V_R^3 in terms of six phase voltages $V_{S'}^6$, $V_{R'}^6$ and current I_p^6 [10] as follows :

$$V_S^3 = \frac{1}{2} N^T V_{S'}^6 + \frac{1}{2} N^T [Z_1] I_p^6 \qquad (3.28)$$

$$V_R^3 = \frac{1}{2} N^T V_{R'}^6 - \frac{1}{2} N^T [Z_2] I_p^6 \qquad (3.29)$$

where $[Z_1]$ and $[Z_2]$ are (6x6) diagonal matrices representing leakage impedances of the transformers TR_1 and TR_2 and [11]

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

In addition, the terminal relationship for a wye/star transformer shown in Fig. 3.2 may be written as

$$V_p^6 = V_{p'}^6 - [Z] I_p^6 \qquad (3.30)$$

Subtracting eqn. (3.29) from eqn. (3.28), we obtain

$$V_S^3 - V_R^3 = \frac{1}{2} N^T (V_{S'}^6 - V_{R'}^6) + \frac{1}{2} N^T [Z_1 + Z_2] I_p^6 \qquad (3.31)$$

Using eqn. (3.27) and the relationship $I_p^6 = NI_p^3$ [11], we get

$$V_S^3 - V_R^3 = \frac{1}{2} N^T [Z_p^6 + Z_1 + Z_2] NI_p^3 \tag{3.32}$$

Hence, from eqn. (3.32), we obtain

$$Z_{p,eq}^3 = \frac{1}{2} N^T [Z_p^6 + Z_1 + Z_2] N \tag{3.33}$$

The equivalent three phase admittance matrix can either be obtained by inverting $Z_{p,eq}^3$ or derived as follows :

From eqn. (3.27) we get

$$I_p^6 = Y_p^6 [V_{S'}^6 - V_{R'}^6] \tag{3.34}$$

Substituting the values of $V_{S'}^6$ and $V_{R'}^6$ from eqns. (3.28) and (3.29) respectively in eqn. (3.34), we obtain

$$I_p^6 = Y_p^6 N[V_S^3 - V_R^3] + Y_p^6 [Z_1+Z_2] NI_p^3 \tag{3.35}$$

We also have [11]

$$I_S^3 = \frac{1}{2} N^T I_S^6 \tag{3.36}$$

Using eqn. (3.35) we get

$$Y_{p,eq}^3 = [U + \frac{1}{2} N^T Y_p^6 \left\{[Z_1]+[Z_2]\right\} N]^{-1} [\frac{1}{2} N^T Y_p^6 N] \tag{3.37}$$

where U is a (3x3) identity matrix .

In a similar fashion, we can derive the three phase equivalent representations when other types of transformers are used.

The three phase equivalent representation based upon ABCD parameters can also be derived. From Fig. 3.7 we get the terminal relationships at bus bars S, S', R and R' as shown

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} N^T & \frac{1}{2} N^T[Z_1] \\ 0 & \frac{1}{2} N^T \end{bmatrix} \begin{bmatrix} V^6_{pS'} \\ I^6_{pS'} \end{bmatrix} \tag{3.38}$$

and

$$\begin{bmatrix} V^6_{pR'} \\ I^6_{pR'} \end{bmatrix} = \begin{bmatrix} N & [Z_2]N \\ 0 & N \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{3.39}$$

Using eqns. (3.26), (3.38) and (3.39) we can write

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} N^T & \frac{1}{2} N^T[Z_1] \\ 0 & \frac{1}{2} N^T \end{bmatrix} \begin{bmatrix} A^6 & B^6 \\ C^6 & D^6 \end{bmatrix} \begin{bmatrix} N & [Z_2]N \\ 0 & N \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{3.40}$$

$$\begin{bmatrix} V^3_{pS} \\ I^3_{pS} \end{bmatrix} = \begin{bmatrix} A^3_{\ell}\ eq & B^3_{\ell}\ eq \\ C^3_{\ell}\ eq & D^3_{\ell}\ eq \end{bmatrix} \begin{bmatrix} V^3_{pR} \\ I^3_{pR} \end{bmatrix} \tag{3.41}$$

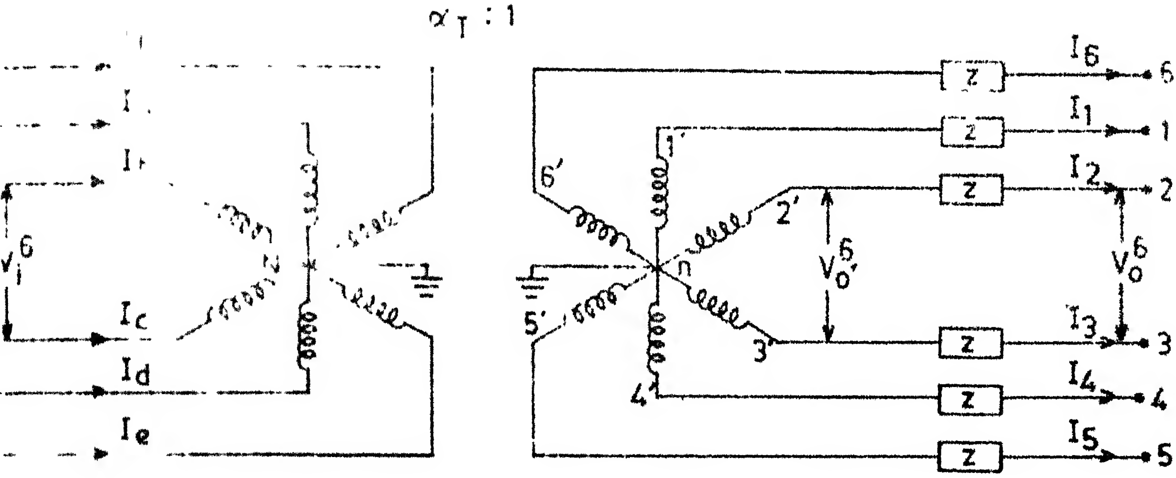


FIG. 3.6 A SIX-PHASE STAR/STAR TRANSFORMER OF TURNS RATIO $\alpha_T : 1$

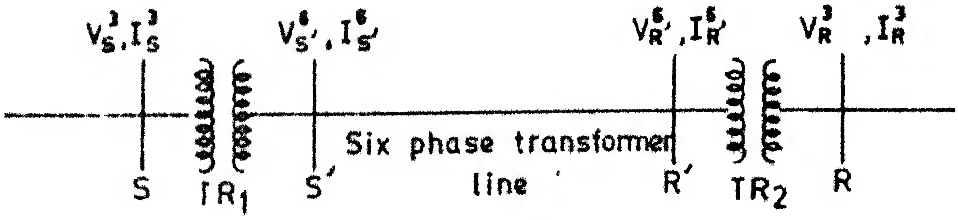


FIG. 3.7 A SIX-PHASE TRANSFORMER LINE CONNECTED BETWEEN TWO THREE PHASE BUSES VIA 3-Ø/6-Ø TRANSFORMERS TR_1 AND TR_2 .

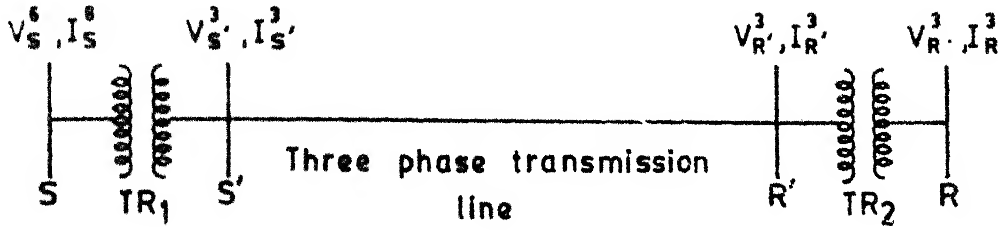


FIG. 3.8 A THREE PHASE TRANSMISSION LINE CONNECTED BETWEEN TWO 6-Ø BUSES THROUGH 6-Ø/3-Ø TRANSFORMERS TR_1 & TR_2.

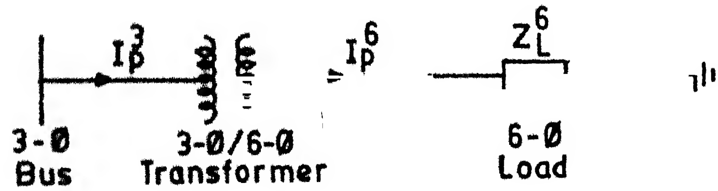


FIG. 3.9 A SIX PHASE LOAD CONNECTED TO THREE PHASE BUSBAR

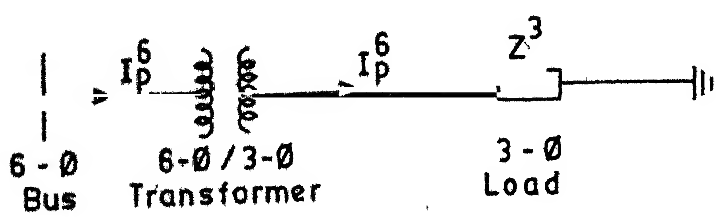


FIG. 3.10 A THREE PHASE LOAD CONNECTED TO SIX PHASE BUSBAR

Noting that $A^6 = D^6$ and A, B, C, D, $[Z_1]$ and $[Z_2]$ are diagonal matrices, eqn. (3.41) after simplification gives

$$A^3_{eq} = \frac{1}{2} N^T A^6 N + \frac{1}{2} N^T [Z_1] C^6 N$$

$$B^3_{eq} = \frac{1}{2} N^T B^6 N + \frac{1}{2} N^T A^6 [Z_1+Z_2] N + \frac{1}{2} N^T [Z_1] C^6 [Z_2] N$$

$$C^3_{eq} = \frac{1}{2} N^T C^6 N$$

$$D^3_{eq} = \frac{1}{2} N^T D^6 N + \frac{1}{2} N^T C^6 [Z_2] N$$

e) Equivalent six phase representation of a three phase line :

If we are interested in the analysis of the six phase part of a composite three phase/six phase power system, an equivalent six phase representation of the three phase part of it becomes essential. However, the analysis on equivalent six phase basis may become tedious because of prohibitive memory requirements.

Consider a system having a three phase line connected to six phase buses via six phase/three phase transformers TR_1 and TR_2 as shown in Fig. 3.8. The six phase equivalent impedance and admittance may be derived [11] in a similar fashion as done earlier in subhead (d). The final results are :

$$Z^6_{p,eq} = \frac{1}{2} N Z^3_p N^T + [Z_1+Z_2] \qquad (3.42)$$

$$Y^6_{p,eq} = [U + \frac{1}{2} N Y^3_p N^T [Z_1+Z_2]]^{-1} \frac{1}{2} N Y^3_p N^T \qquad (3.43)$$

Noting that $A^6 = D^6$ and A, B, C, D, $[Z_1]$ and $[Z_2]$ are diagonal matrices, eqn. (3.41) after simplification gives

$$A^3_{,eq} = \frac{1}{2} N^T A^6 N + \frac{1}{2} N^T [Z_1] C^6 N$$

$$B^3_{,eq} = \frac{1}{2} N^T B^6 N + \frac{1}{2} N^T A^6 [Z_1+Z_2] N + \frac{1}{2} N^T [Z_1] C^6 [Z_2] N$$

$$C^3_{,eq} = \frac{1}{2} N^T C^6 N$$

$$D^3_{,eq} = \frac{1}{2} N^T D^6 N + \frac{1}{2} N^T C^6 [Z_2] N$$

e) Equivalent six phase representation of a three phase line :

If we are interested in the analysis of the six phase part of a composite three phase/six phase power system, an equivalent six phase representation of the three phase part of it becomes essential. However, the analysis on equivalent six phase basis may become tedious because of prohibitive memory requirements.

Consider a system having a three phase line connected to six phase buses via six phase/three phase transformers TR_1 and TR_2 as shown in Fig. 3.8. The six phase equivalent impedance and admittance may be derived [11] in a similar fashion as done earlier in subhead (d). The final results are :

$$Z^6_{p,eq} = \frac{1}{2} N\, Z^3_p\, N^T + [Z_1+Z_2] \qquad (3.42)$$

$$Y^6_{p,eq} = [U + \frac{1}{2} N\, Y^3_p\, N^T [Z_1+Z_2]]^{-1} \frac{1}{2} N\, Y^3_p\, N^T \qquad (3.43)$$

where U is a (6x6) order identity matrix.

Similarly, the equivalent six phase ABCD parameters of the transmission line may be derived as under :

$$A^6_{,eq} = \frac{1}{2} NA^3N^T + \frac{1}{2} [Z_1]NCN^T$$

$$B^6_{,eq} = \frac{1}{2} NB^3N^T + \frac{1}{2} NA^3N^T[Z_2] + \frac{1}{2} [Z_1]ND^3N^T + \frac{1}{2} [Z_1]NC^3N^T[Z_2]$$

$$C^6_{,eq} = \frac{1}{2} NC^3N^T$$

$$D^6_{,eq} = \frac{1}{2} ND^3N^T + \frac{1}{2} NC^3N^T[Z_2]$$

3.6 MODELLING OF SIX PHASE LOADS

Multiphase loads may be modelled as a constant impedance/ admittance to ground. Here again, we have to obtain three phase equivalent representation of a six phase load or vice versa depending upon whether our interest of analysis lies on the three phase part or six phase part of the composite system.

Consider a six phase load connected via a three phase/ six phase wye/star transformer to a three phase bus as shown in Fig. 3.9.

For the six phase load, we get

$$V^6_p = Z^6_p \; I^6_p \tag{3.44}$$

We also get voltage V_p^3 as

$$V_p^3 = \frac{1}{2} N^T V_p^6 + \frac{1}{2} N^T [Z] I_p^6 \tag{3.45}$$

or

$$V_p^3 = \frac{1}{2} N^T Z_L{}^6 N I_p^3 + \frac{1}{2} N^T [Z] N I_p^3 \tag{3.46}$$

Therefore,

$$Z_{L,eq}^3 = \frac{1}{2} N^T [Z_L^6 + Z] N \tag{3.47}$$

Similarly, we can derive the equivalent three phase load admittance matrix as shown below :

$$Y_{L,eq}^3 = [U + \frac{1}{2} N^T Y_L^6 [Z] N]^{-1} [\frac{1}{2} N^T Y_L^6 N] \tag{3.48}$$

Now, consider a three phase load connected to a six phase bus through a six phase/three phase star/wye transformer as shown in Fig. 3.10. The load can be represented as

$$V_p^3 = Z_L^3 I_p^3 \tag{3.49}$$

We obtain voltage V_p^6 as

$$V_p^6 = \frac{1}{2} N Z_L^3 N^T I_p^6 + [Z] I_p^6 \tag{3.50}$$

or

$$Z_{L,eq}^6 = \frac{1}{2} N Z_L^3 N^T + Z \tag{3.51}$$

Similarly, we can derive the six phase equivalent admittance in phase form as shown :

$$Y_{L,eq}^6 = [U + \frac{1}{2} N Y_L^3 N^T Z]^{-1} [\frac{1}{2} N Y_L^3 N^T] \tag{3.52}$$

3.7 SAMPLE NETWORK

The mixed three phase and six phase load flow analysis of the sample network shown in Fig. 3.11 was carried out. The sample system is the same as that of Fig. 2.14 except that the three double circuit three phase lines are replaced by six phase lines. This can be achieved by installing two three-phase/six-phase wye-star transformers at the sending end and at the receiving end respectively of the six phase line. The bus numbering sequence with such an arrangement is shown in Fig. 3.11.

The effect of converting the double circuit three phase line to a six phase line has been studied. For the purpose of study, each double circuit three phase line was converted to a six phase line one by one. Then, two double circuit three phase lines were converted to six phase lines simultaneously and finally, all the three double circuit three phase lines were converted to six phase lines. The necessary data are given in Table 3.3. Gauss-Seidel iterative technique was used to conduct the load flow analysis. The algorithm used was the same as that used in the three phase load flow except that the admittance submatrices for the six phase lines and three phase/six phase transformers were formed separately and the admittance matrix of the system modified accordingly. The flow chart for developing these submatrices is given in the Appendix.

In the first instance, the load flow analysis of the balanced system were conducted for all the cases mentioned above After that, the system was made unbalanced with the introduction of the same unbalances which were simulated in the case of three phase load flow (Chapter II) in order to study their effect on the system when it contains six phase lines. The results obtained from the balanced and unbalanced load flow studies with three six phase lines in the system are shown in Tables 3.4-3.5.

3.8 CONCLUSIONS

The relative merits and demerits of multiphase systems over the conventional three phase systems have been discussed. The detailed mathematical modelling of the six phase elements such as machines, transformers and transmission line has been developed. Three phase equivalent representation of a six phase line and vice versa have also been developed. The following conclusions are drawn from the mixed three phase six phase load flow analysis conducted and results given in Tables 3.4-3.5.

a) When only one double circuit three phase line is converted to a six phase line, the voltage magnitudes at the two buses connecting the six phase line are improved while the voltages at the other buses remain nearly unchanged.

b) The improvement in the voltage magnitudes is more when two double circuit three phase lines are converted to six phase lines and the results are best when all the three double circuit three phase lines are six phase lines.

c) The voltage angles at the buses near the generators are reduced slightly.

d) As a consequence of reduction of voltage angles, the reactive power flows from the generators are reduced slightly.

e) The real power flow from the slack generator increases slightly.

f) It was found, that, with the same unbalances introduced as in the three phase case, the degree of unbalance in the case of mixed three phase six phase load flow remains same. In fact, there is a slight improvement in the voltage magnitudes unbalancing.

The above para give the advantages of the six phase transmission system over the conventional three phase systems.

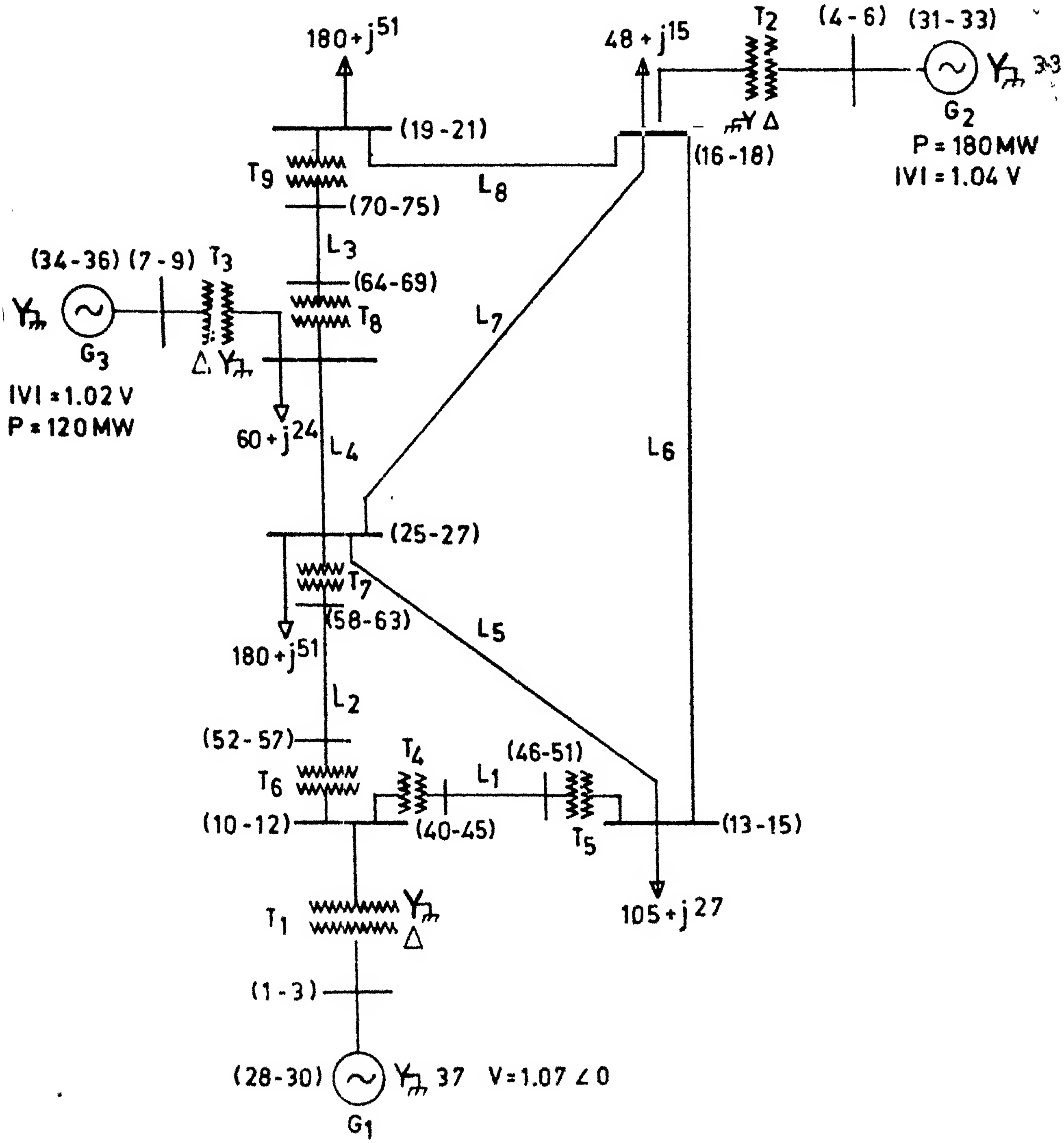


FIG. 3.11 THE SAMPLE NETWORK WITH BUS NUMBERING SEQUENCE FOR MIXED 3-Ø AND 6-Ø LOAD FLOW STUDY.

Table 3.3

Circuit characteristics of the network of Fig. 3.11 on 100 MVA/phase base and rate KV

From Bus	To Bus	1,2,3,4,5 sequence			Zero sequence		
		R	X	B	R_o	X_o	B_o
1) 6-Ø Lines :							
(40-45)	(46-51)	0.0125	0.030	0.01875	0.03875	0.145	0.010875
(52-57)	(58-63)	0.00625	0.025	0.00938	0.01934	0.0725	0.0054
(64-69)	(70-75)	0.0083	0.033	0.01245	0.0258	0.0966	0.0072

2) 3-Ø/6-Ø Transformers :

The transformers T_4-T_9 are wye-star solidly earthed with

$y_{PS} = y_{PT} = y_{ST} = -j12.5$

(Rest of the data is same as that in Table 2.4)

Table 3.4

Voltage solutions for mixed three phase six phase load flow for the network of Fig. 3.11

Bus No.	Balanced Case $\lvert V\rvert$ in p.u.	Balanced Case θ in deg.	Unbalanced Case $\lvert V\rvert$ in p.u.	Unbalanced Case θ in deg.
1	1.043	-2.731	1.039	-5.111
2	1.043	-122.799	1.034	-124.712
3	1.043	118.335	1.044	115.446
4	1.027	-1.449	1.032	89.588
5	1.028	-121.504	1.016	-28.871
6	1.028	+119.575	1.040	-148.857
7	1.006	-3.497	1.003	-5.995
8	1.007	-121.554	0.998	-125.479
9	1.006	117.525	1.009	114.616
10	1.031	82.518	1.024	82.862
11	1.032	-37.411	1.034	-37.700
12	1.031	-157.466	1.017	-157.840
13	1.026	81.927	1.020	82.257
14	1.027	-38.002	1.032	-38.309
15	1.027	-158.056	1.014	-158.412
16	1.024	83.599	1.016	84.913
17	1.024	-36.332	1.020	-36.902
18	1.025	-156.365	0.988	-157.115
19	0.993	80.377	0.987	81.016
20	0.993	-39.558	1.000	-39.921
21	0.994	-159.588	0.973	-160.294
22	0.997	80.821	0.991	81.382
23	0.997	-39.109	1.003	-39.458
24	0.997	-159.145	0.979	-159.789
25	1.022	81.784	1.017	82.182
26	1.023	-38.145	1.028	-38.454
27	1.023	-158.198	1.009	-158.631
28	1.070	0.000	1.070	0.000
29	1.070	-120.072	1.070	-120.072
30	1.070	120.072	1.070	120.072
31	1.040	2.995	1.040	96.102
32	1.041	-117.062	1.041	-23.955
33	1.041	123.003	1.041	-143.968
34	1.020	-1.729	1.020	-1.829
35	1.021	-121.787	1.021	-121.886
36	1.021	118.278	1.021	118.179
37	0.000	0.000	0.000	0.000
38	0.000	0.000	0.000	0.000
39	0.000	0.000	0.000	0.000
40	1.782	82.432	1.773	82.774

Bus No.	Balanced Case $\|V\|$ in p.u.	 θ in deg.	Unbalanced Case $\|V\|$ in p.u.	 θ in deg.
41	1.782	22.449	1.761	22.076
42	1.782	-37.496	1.791	-37.787
43	1.782	-97.567	1.773	-97.226
44	1.783	-157.551	1.761	-157.923
45	1.782	142.504	1.791	142.214
46	1.778	82.012	1.768	82.346
47	1.778	22.029	1.756	21.672
48	1.778	-37.917	1.788	-38.222
49	1.778	-97.988	1.768	-97.654
50	1.778	-157.970	1.756	-158.327
51	1.778	142.084	1.788	141.779
52	1.726	80.734	1.716	81.308
53	1.726	20.768	1.694	20.109
54	1.726	-39.196	1.737	-39.548
55	1.726	-99.265	1.716	-98.691
56	1.726	-159.231	1.694	-159.890
57	1.726	140.805	1.737	140.453
58	1.721	80.464	1.711	81.089
59	1.722	20.499	1.687	19.808
60	1.721	-39.466	1.733	-39.831
61	1.721	-99.535	1.711	-98.910
62	1.722	-159.501	1.687	-160.191
63	1.721	140.535	1.733	140.170
64	1.780	82.348	1.771	82.703
65	1.781	22.364	1.759	21.975
66	1.781	-37.581	1.789	-37.874
67	1.780	-97.652	1.771	-97.297
68	1.781	-157.636	1.759	-158.025
69	1.781	142.419	1.789	142.127
70	1.772	81.955	1.763	82.342
71	1.773	21.973	1.750	21.556
72	1.773	-37.974	1.782	-38.279
73	1.772	-98.045	1.763	-97.657
74	1.773	-158.027	1.750	-158.444
75	1.773	142.027	1.782	141.721

Table 3.5

Power Flows

To Bus	From Bus	Balanced Case Real Power (MW)	Balanced Case Reactive (MVAR)	Unbalanced Case Real Power (MW)	Unbalanced Case Reactive Power (MVAR)
1	28	95.102	31.551	102.490	38.926
2	29	95.032	31.350	92.591	43.812
3	30	95.229	31.552	93.213	32.891
4	31	59.675	10.536	70.863	9.017
5	32	59.759	10.715	53.368	17.509
6	38	59.546	10.642	54.743	-1.960
7	34	39.687	9.802	43.763	11.793
8	35	39.749	9.968	37.576	15.258
9	36	39.564	9.899	37.663	8.124
25	22	-11.050	-11.561	-9.725	-12.321
26	23	-11.100	-11.716	-11.790	-11.070
27	24	-10.948	-11.686	-12.440	-12.985
25	13	0.846	-2.715	0.609	-2.611
26	14	0.849	-2.710	0.894	-2.864
27	15	0.841	-2.712	1.081	-2.496
16	13	-4.727	-3.718	-7.290	-2.103
17	14	-4.707	-3.656	-3.968	-4.851
18	15	-4.775	-3.671	-2.791	-0.273
25	16	7.965	-4.792	11.650	-6.681
26	17	7.937	-4.883	6.869	-3.424
27	18	8.034	-4.860	5.276	-9.352
19	16	30.865	7.291	35.958	4.766
20	17	30.854	7.271	29.812	9.287
21	18	30.892	7.283	27.061	-0.026
40	46	-13.979	-1.612	-14.331	-2.599
41	47	-13.967	-1.637	-13.423	-2.740
42	48	-13.989	-1.637	-14.442	-0.928
43	49	-13.979	-1.611	-14.330	-2.599
44	50	-13.967	-1.636	-13.423	-2.739
45	51	-13.988	-1.636	-14.441	-0.927
64	70	-27.879	-9.160	-25.827	-10.356
65	71	-27.818	-9.247	-29.648	-12.017

To Bus	From Bus	Balanced Case Real Power (MW)	Balanced Case Reactive Power (MVAR)	Unbalanced Case Real Power (MW)	Unbalanced Case Reactive Power (MVAR)
66	72	-27.910	-9.267	-28.772	-8.187
67	73	-27.879	-9.159	-25.826	-10.356
68	74	-27.817	-9.247	-29.648	-12.016
69	75	-27.910	-9.267	-28.771	-8.186
52	58	-13.347	-4.118	-11.197	-5.378
53	59	-13.345	-4.127	-15.037	-7.184
54	60	-13.349	-4.123	-13.919	-3.095
55	61	-13.347	-4.118	-11.197	-5.378
56	62	-13.345	-4.126	-15.037	-7.184
57	63	-13.349	-4.123	-13.919	-3.095

CHAPTER 4

TWELVE PHASE POWER SYSTEM: MODELLING

4.1 INTRODUCTION

As the demand for electric power is increasing at an exponential rate, the emphasis of the scientists and power system engineers has been to develop feasible alternatives of transmitting more power from generating stations to the load centres. One of the striking alternatives, as pointed out earlier, is the use of high phase order transmission.

A considerable amount of work has been done on the feasibility of six phase systems, yet the topic of twelve phase systems seems to have been left untouched, in this respect. Although the idea of using twelve phase systems is in its infancy, one should not wonder if it becomes a reality in future. To make twelve phase systems operative, careful planning of such system is required. Planning studies can be done only if suitable mathematical modelling of the twelve phase systems is available.

This chapter presents the mathematical modelling of the different elements present in a twelve-phase power system network. In the first part, the twelve phase synchronous machine equation incorporating all the unbalances has been developed. After that, the nodal relationships between

current and voltage for a three-phase/twelve phase transformer has been derived. This is followed by the presentation of admittance matrix of a twelve phase transmission line based upon the phase impedance matrix, π circuit representation and ABCD parameters. Three phase equivalent representation of a twelve phase line and vice versa have also been derived. Finally, equations have been developed to model twelve-phase loads.

4.2 MODELLING OF TWELVE-PHASE GENERATOR

Although the feasibility of a twelve-phase generator is yet to be established (even six-phase generators have not yet come into existance), a model similar to that of three and six-phase synchronous generators has been developed in this section.

The schematic representation of a general twelve-phase element is given in Fig. 4.1. The figure represents a star-connected twelve-phase synchronous machine if $E_a, E_b, \ldots, E_l$ are a balanced set of internal voltages and nodes 13,14,...,24 are joined together to form a star point. Hence, a twelve phase synchronous generator is represented by a balanced set of internal voltages behind proper reactances. The equation of the element on Fig. 4.1 in the impedance form is

$$V_{p'}^{12} + E_p^{12} = Z_p^{12} I_{p'}^{12} + V_p^{12} \tag{4.1}$$

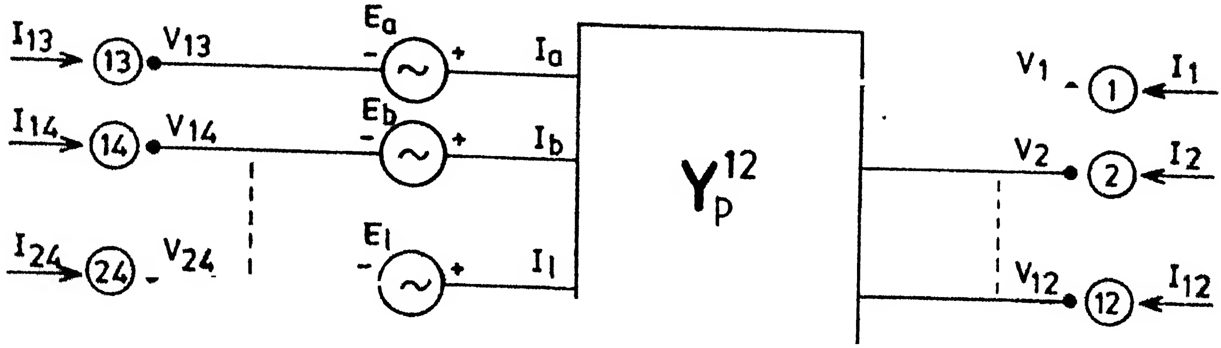


FIG. 4.1 SCHEMATIC REPRESENTATION OF A GENERAL 12-Ø ELEMENT.

where

$$V_{p'}^{12} = [V_{13}\ V_{14}\ \cdots\ V_{24}]^T$$

$$E_p^{12} = [E_a\ \ E_b\ \ \cdots\ \ E_l]^T$$

$$V_p^{12} = [V_1\ \ V_2\ \ \cdots\ \ V_{12}]^T$$

$$I_{p'}^{12} = [I_{13}\ I_{14}\ \cdots\ I_{24}]^T$$

$$= [I_a\ \ I_b\ \ \cdots\ \ I_l]^T$$

and $\quad Z_p^{12}$ = (12x12) impedance matrix of the 12-phase element

$$= [Y_p^{12}]^{-1}$$

From eqn. (4.1), we can write the nodal equations of Fig. 4.1 for the voltage and current directions shown as follows :

$$\begin{bmatrix} I_p^{12} \\ \\ I_{p'}^{12} \end{bmatrix} = \begin{bmatrix} Y_p^{12} & -Y_p^{12} & -Y_p^{12} \\ \\ -Y_p^{12} & Y_p^{12} & Y_p^{12} \end{bmatrix} \begin{bmatrix} V_p^{12} \\ V_{p'}^{12} \\ E_p^{12} \end{bmatrix} \tag{4.2}$$

where, $I_p^{12} = [I_1\ \ I_2\ \ \cdots\ \ I_{12}]^T$

For a balanced set of internal voltages,

$$E_p^{12} = [1\ \ \alpha^*\ \ \alpha^{2*}\ \ -j\ \ -\alpha^2\ \ -\alpha\ \ -1\ \ -\alpha^*\ \ -\alpha^{2*}\ \ j\ \ \alpha^2\ \ \alpha]^T\ E_a \tag{4.3}$$

where, $\alpha = e^{j2\pi/12} = 0.866+j0.5 = 12\sqrt{1} = 1\underline{/30^\circ}$

If the nodes 13-24 are joined together, then

$$I_N = I_{13}+I_{14} + \dots + I_{24} \tag{4.4}$$

and

$$V_N = V_{13} = V_{14} = \dots = V_{24} \tag{4.5}$$

Thus internal voltages are given by,

$$E_p^{12} = [E_a+V_N,\ E_b+V_N,\ \dots,\ E_l+V_N]^T \tag{4.6}$$

and the current I_p^{12}, by

$$I_a = \frac{S_a^*}{(V_N+E_a)^*}\ ;\ I_b = \frac{S_b^*}{(V_N+E_b)^*}\ ;\ \dots\ ;\ I_l = \frac{S_l^*}{(V_N+E_l)^*} \tag{4.7}$$

where $S_a, S_b, \dots, S_l$ are complex powers for phases a,b,...,l respectively.

The (12x12) admittance matrix Y_p^{12} in eqn. (4.2) is given by,

$$\begin{bmatrix} Y_s & Y_{m1} & Y_{m2} & Y_{m3} & \cdots & Y_{m11} \\ Y_{m11} & Y_s & Y_{m1} & Y_{m2} & \cdots & Y_{m10} \\ Y_{m10} & Y_{m11} & Y_s & Y_{m1} & \cdots & Y_{m9} \\ & & \vdots & & & \\ Y_{m1} & Y_{m2} & Y_{m3} & Y_{m4} & \cdots & Y_s \end{bmatrix} \tag{4.8}$$

The expression for Y_p^{12} is obtained after using symmetrical component twelve-phase transformation matrix T_s^{12} [24]

i.e., $$Y_p^{12} = [T_s^{12}][Y_{comp}^{12}][T_s^{12^*}]^T \tag{4.9}$$

where,

$$T_s^{12} = \frac{1}{\sqrt{12}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 & j & -\alpha^{2*} & -\alpha^* & -1 & -\alpha & -\alpha^2 & -j & \alpha^{2*} & \alpha^* \\ 1 & \alpha^2 & -\alpha^{2*} & -1 & -\alpha^2 & \alpha^{2*} & 1 & \alpha^2 & -\alpha^{2*} & -1 & -\alpha^2 & \alpha^{2*} \\ 1 & j & -1 & -j & 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & -\alpha^{2*} & -\alpha^2 & 1 & -\alpha^{2*} & -\alpha^2 & 1 & -\alpha^{2*} & -\alpha^2 & 1 & -\alpha^{2*} & -\alpha^2 \\ 1 & -\alpha^* & \alpha^{2*} & j & -\alpha^2 & \alpha & -\alpha & \alpha^* & -\alpha^{2*} & -j & \alpha^2 & -\alpha \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & -\alpha & \alpha^2 & -j & -\alpha^{2*} & \alpha^* & -1 & \alpha & -\alpha^2 & j & -\alpha^{2*} & -\alpha^* \\ 1 & -\alpha^2 & -\alpha^{2*} & 1 & -\alpha^2 & -\alpha^{2*} & 1 & -\alpha^2 & -\alpha^{2*} & 1 & -\alpha^2 & -\alpha^{2*} \\ 1 & -j & -1 & j & 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \alpha^{2*} & -\alpha^2 & -1 & -\alpha^{2*} & \alpha^2 & 1 & \alpha^{2*} & -\alpha^2 & -1 & -\alpha^{2*} & \alpha^2 \\ 1 & \alpha^* & \alpha^{2*} & -j & -\alpha^2 & -\alpha & -1 & -\alpha^* & -\alpha^{2*} & j & \alpha^2 & \alpha \end{bmatrix}$$

and $$Y_{comp}^{12} = \text{diag.}\ [y_o,\ y_1,\ y_2,\ \ldots,\ y_{11}] \tag{4.11}$$

$y_o, y_1, y_2, \ldots\ y_{11}$ are twelve-phase sequence admittances.

After using eqn. (4.9), various elements of the eqn. (4.8) are obtained as given below :

$$Y_s = \frac{1}{12}(y_o+y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9+y_{10}+y_{11})$$

$$Y_{m1} = \frac{1}{12}(y_o+\alpha y_1+\alpha^2 y_2+jy_3-\alpha^{2*}y_4-\alpha^* y_5-y_6-\alpha y_7-\alpha^2 y_8-jy_9+\alpha^{2*}y_{10}+\alpha^* y_{11})$$

$$Y_{m2} = \frac{1}{12}(y_o+\alpha^2 y_1-\alpha^{2*}y_2-y_3-\alpha^2 y_4+\alpha^{2*}y_5+y_6+\alpha^2 y_7-\alpha^{2*}y_8$$
$$-y_9-\alpha^2 y_{10}+\alpha^{2*}y_{11})$$

$$Y_{m3} = \frac{1}{12}(y_o+jy_1-y_2-jy_3+y_4+jy_5-y_6-jy_7+y_8+jy_9-y_{10}-jy_{11})$$

$$Y_{m4} = \frac{1}{12}(y_o-\alpha^{2*}y_1-\alpha^2 y_2+y_3-\alpha^{2*}y_4-\alpha^2 y_5+y_6-\alpha^{2*}y_7-\alpha^2 y_8+y_9-\alpha^{2*}y_{10}$$
$$-\alpha^2 y_{11})$$

$$Y_{m5} = \frac{1}{12}(y_o-\alpha^* y_1+\alpha^{2*}y_2+jy_3-\alpha^2 y_4+\alpha y_5-y_6+\alpha^* y_7-\alpha^{2*}y_8-jy_9$$
$$+\alpha^2 y_{10}-\alpha y_{11})$$

$$Y_{m6} = \frac{1}{12}(y_o-y_1+y_2-y_3+y_4-y_5+y_6-y_7+y_8-y_9+y_{10}-y_{11})$$

$$Y_{m7} = \frac{1}{12}(y_o-\alpha y_1+\alpha^2 y_2-jy_3-\alpha^{2*}y_4+\alpha^* y_5-\alpha^* y_6+\alpha y_7-\alpha^2 y_8+jy_9$$
$$+\alpha^{2*}y_{10}-\alpha^* y_{11})$$

$$Y_{m8} = \frac{1}{12}(y_o-\alpha^2 y_1-\alpha^{2*}y_2+y_3-\alpha^2 y_4-\alpha^{2*}y_5+y_6-\alpha^2 y_7-\alpha^{2*}y_8$$
$$+y_9-\alpha^2 y_{10}-\alpha^{2*}y_{11})$$

$$Y_{m9} = \frac{1}{12}\,(y_o - jy_1 - y_2 + jy_3 + y_4 - jy_5 - y_6 + jy_7 + y_8 - jy_9 - y_{10} + jy_{11})$$

$$Y_{m10} = \frac{1}{12}\,(y_o + \alpha^{2*}y_1 - \alpha^2 y_2 - y_3 - \alpha^{2*}y_4 + \alpha^2 y_5 + y_6 + \alpha^{2*}y_7 - \alpha^2 y_8 - y_9 - \alpha^{2*}y_{10} + \alpha^2 y_{11})$$

$$Y_{m11} = \frac{1}{12}\,(y_o + \alpha^{*}\,y_1 + \alpha^{2*}y_2 - jy_3 - \alpha^2 y_4 - \alpha y_5 - y_6 - \alpha^{*}y_7 - \alpha^{2*}y_8 + jy_9 + \alpha^2 y_{10} + \alpha y_1)$$

Substituting the values of E_p^{12}, I_p^{12}, and Y_p^{12} from eqns. (4.6), (4.7) and (4.8) respectively in eqn. (4.2), we obtain after simplification and collection of terms

$$\left[\begin{array}{ccc}
 & -y_o & -y_1 \\
 & -y_o & -\alpha y_1 \\
 & -y_o & -\alpha^2 y_1 \\
 & -y_o & -jy_1 \\
Y_p^{12} & -y_o & -\alpha^{2*} y_1 \\
 & -y_o & \alpha^{*} y_1 \\
 & -y_o & y_1 \\
 & -y_o & \alpha y_1 \\
 & -y_o & \alpha^2 y_1 \\
 & -y_o & jy_1 \\
 & -y_o & -\alpha^{2*} y_1 \\
 & -y_o & -\alpha^{*} y_1
\end{array}\right]
\Bigg[\; V_p^{12} \;\Bigg] = \Bigg[\; I_p^{12} \;\Bigg]$$

$$\begin{bmatrix} & y_o & y_1 \\ & y_o & \alpha y_1 \\ & y_o & \alpha^2 y_1 \\ & y_o & j y_1 \\ & y_o & -\alpha^{2*} y_1 \\ -Y_p^{12} & y_o & -\alpha^* y_1 \\ & y_o & -y_1 \\ & y_o & -\alpha y_1 \\ & y_o & -\alpha^2 y_1 \\ & y_o & -j y_1 \\ & y_o & \alpha^{2*} y_1 \\ & y_o & \alpha^* y_1 \end{bmatrix} \begin{bmatrix} \\ V_N \\ E_a \\ \end{bmatrix} = \begin{bmatrix} S_a^*/(V_N+E_a)^* \\ S_b^*/(V_N+E_b)^* \\ \vdots \\ S_1^*/(V_N+E_1)^* \end{bmatrix} \tag{4.12}$$

By adding the last twelve eqns. of (4.12), we get an additional equation for node N as

$$I_N = [-y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ y_o\ 12y_o\ 0] \begin{bmatrix} V_p^{12} \\ V_N \\ E_a \end{bmatrix} \tag{4.13}$$

It can be deduced by taking the conjugate of eqn. (4.3) that

$$E_p^{12^*} = [1\ {=}\alpha\ \alpha^2\ j\ -\alpha^{2*}\ -\alpha^*\ -1\ -\alpha\ -\alpha^2\ -j\ \alpha^{2*}\ \alpha^*]^T E_a^* \tag{4.14}$$

Crossmultiplication of the last twelve equations of (4.12), their addition and simplification of the expression gives the following result

$$-y_oV_1V_N^* \quad -y_oV_2V_N^* \quad -y_oV_3V_N^* \quad -y_oV_4V_N^* \quad -y_oV_5V_N^* \quad -y_oV_6V_N^*$$

$$-y_oV_7V_N^* \quad -y_oV_8V_N^* \quad -y_oV_9V_N^* \quad -y_oV_{10}V_N^* \; -y_oV_{11}V_N^* \; -y_oV_{12}V_N^*$$

$$+12y_oV_NV_N^* \; -y_1V_1E_a^* \quad -y_1\alpha^*V_2E_a^* \; -\alpha^{2*}y_1V_3E_a^*+jy_1V_4E_a^*$$

$$+\alpha^2y_1V_5E_a^* \; +\alpha y_1V_6E_a^* \; +y_1V_7E_a^* + \alpha^*V_8E_a^* + \alpha^{2*}y_1V_9E_a^*$$

$$-jy_1V_{10}E_a^* \; -\alpha^2y_1V_{11}E_a^* \; -\alpha y_1V_{12}E_a^* + 12y_1E_aE_a^*$$

$$= S_a^* + S_b^* + \ldots + S_1^* \tag{4.15}$$

Noting that the first thirteen terms of eqn. (4.15) are V_N^* times I_N and the injected neutral current $I_N = 0$, we can write eqn. (4.15) as

$$[-y_1 \; -\alpha^* y_1 \; -\alpha^{2*}y_1 \; jy_1 \; \alpha^2y_1 \; \alpha y_1 \; y_1 \; \alpha^*y_1 \; \alpha^{2*}y_1 \; -jy_1 \; -\alpha^2y_1 \; -\alpha y_1]$$

$$\begin{bmatrix} V_p^{12} \\ \\ E_a \end{bmatrix} = \frac{S_a^*+S_b^*+ \ldots + S_1^*}{E_a^*} \tag{4.16}$$

If the neutral is earthed through an admittance y_{No}, eqn.(4.4) is modified to

$$I_N = I_{13}+I_{14}+ \ldots + I_{24} + y_{No}\, V_N \tag{4.17}$$

Thus, from eqn. (4.13), we obtain

$$I_N = [-y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ -y_o\ 12y_o + y_{No}] \begin{bmatrix} V_p^{12} \\ V_N \end{bmatrix} \tag{4.18}$$

Substituting the value of $V_N^* I_N$ from eqn. (4.18) in eqn. (4.15), we get with $I_N = 0$, the following

$$[-y_1\ -\alpha^* y_1\ -\alpha^{2*} y_1\ jy_1\ \alpha^2 y_1\ \alpha y_1\ y_1\ \alpha^* y_1\ \alpha^{2*} y_1\ -jy_1\ -\alpha^2 y_1\ -\alpha y_1] \begin{bmatrix} V_p^{12} \\ E_a \end{bmatrix} = \frac{\Sigma S^*}{E_a^*} + \frac{y_{No}|V_N|^2}{E_a^*} \tag{4.19}$$

where $\Sigma S^* = S_a^* + S_b^* + \dots + S_l^*$

Neglecting the second term in the R.H.S. of eqn. (4.19), the final equation of a twelve phase synchronous generator with all the types of unbalances included in it is obtained by collecting the first twelve equations of (4.12), eqn. (4.18) and eqn. (4.19). The concise form of the final eqn. is

$$\begin{bmatrix} Y_p^{12} & -\bar{Y}_1 & -\bar{Y}_o \\ -\bar{Y}_2 & 12y_1 & 0 \\ -\bar{Y}_o^T & 0 & 12y_o+y_{No} \end{bmatrix} \begin{bmatrix} V_p^{12} \\ E_a \\ V_N \end{bmatrix} = \begin{bmatrix} I_p^{12} \\ \Sigma S^*/E_a^* \\ 0 \end{bmatrix} \tag{4.20}$$

where,

$$\bar{Y}_1 = =[1\ \alpha\ \alpha^2\ j\ -\alpha^{2*}\ -\alpha^*\ -1\ -\alpha\ -\alpha^2\ -j\ \alpha^{2*}\ \alpha^*]^T\ y_1\ ,$$

$$\bar{Y}_2 = [1\ \alpha^*\ \alpha^{2*}\ -j\ -\alpha^2\ -\alpha\ -1\ -\alpha^*\ -\alpha^{2*}\ j\ \alpha^2\ \alpha]^T\ y_1\ ,$$

$$\bar{Y}_o = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]^T\ y_o$$

where $\alpha = e^{j2\pi/12} = 1\angle 30^o$.

Eqn. (4.20) is the complete eqn. of a twelve phase machine capable of including all the unbalances present in it.

4.3 MODELLING OF TWELVE PHASE TRANSFORMERS

Transformers used for multiphase systems are required for obtaining higher phase order conversion from a given three phase system. A twelve phase conversion can be obtained by using wye/star or delta/star three phase/twelve phase transformers. The feasibility of a twelve phase transformer and the procedure to obtain twelve phase connection from a three phase transformer has been given by Uma Pal et al. [24]. There may be other ways of obtaining twelve phase conversion also but in the present work, the models of the two types of transformers mentioned above have been developed.

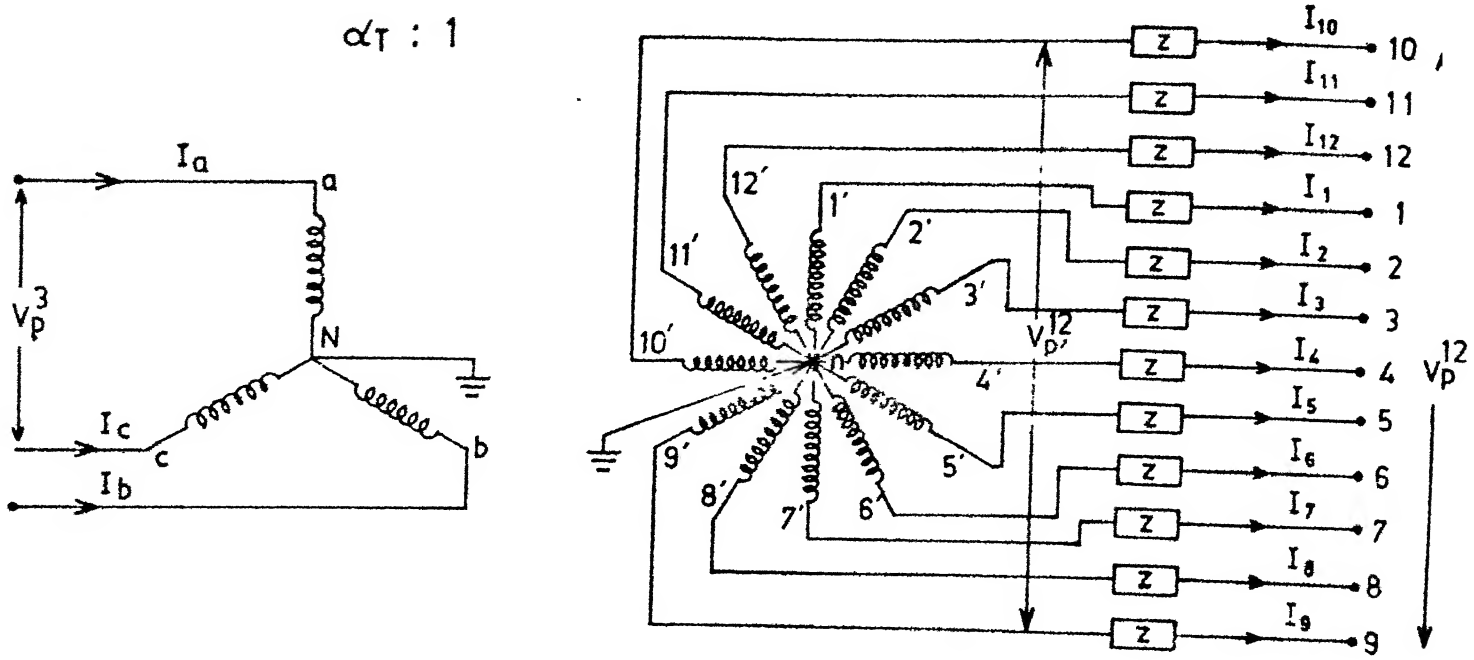


FIG. 4.2 SCHEMATIC REPRESENTATION OF A THREE PHASE / TWELVE PHASE, WYE / STAR TRANSFORMER WITH TURNS RATIO $\alpha_T : 1$

The transformers are represented as ideal transformers in series with the equivalent p.u. leakage impedances of the windings. Consider a wye/star connected, three phase/twelve phase transformer shown in Fig. 4.2. A nominal ratio transformer is represented by an ideal transformer in series with the p.u. leakage impedance Z of the windings. The voltage and current relations at the terminals of the ideal transformer are :

$$V_{p'}^{12} = [M]V_p^3 \; ; \qquad V_p^3 = \frac{1}{8}\,[L]\,V_{p'}^{12} \tag{4.21}$$

$$I_{p'}^{12} = [M]I_p^3 \; ; \qquad I_p^3 = \frac{1}{8}\,[L]\,I_{p'}^{12} \tag{4.22}$$

where,

$$V_p^3 = [V_a \; V_b \; V_c]^T,$$

$$V_{p'}^{12} = [V_1 \; V_2 \; \dots \; V_{12}]^T,$$

$$I_p^3 = [I_a \; I_b \; I_c]^T,$$

$$I_p^{12} = [I_1 \; I_2 \; \dots \; I_{12}]^T,$$

$$Z = (12\text{x}12) \text{ order diagonal matrix} = [y]^{-1}$$

$$\text{and } M^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The terminal voltages of the transformer in temms of the induced voltage vector V_p^{12} on the 12-phase side is given by

$$V_p^{12} = V_{p'}^{12} - [Z]\, I_p^{12} \tag{4.23}$$

The matrix L is given by

$$L = \begin{bmatrix} 2 & 1 & -1 & 0 & 1 & -1 & -2 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 & 2 & 1 & -1 & 0 & 1 & -1 & -2 & -1 \\ 1 & -1 & -2 & -1 & 1 & 0 & -1 & 1 & 2 & 1 & -1 & 0 \end{bmatrix}$$

From eqns. (4.21) and (4.23), we obtain

$$I_p^{12} = [y][M]V_p^3 - [y]V_p^{12} \tag{4.24}$$

Also from eqns. (4.22) and (4.24), we get

$$I_p^3 = \frac{1}{8}\,[L]\; y[M]V_p^3 - \frac{1}{8}\,[L]\; y\; V_p^{12} \tag{4.25}$$

Thus the nodal representation of a wye/star, three phase/ twelve phase transformer may be written as follows :

$$\begin{bmatrix} I_p^3 \\ -I_p^{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{8}\,[L]\,[y][M] & -\frac{1}{8}\,[L]\,[y] \\ -[y][M] & [y] \end{bmatrix} \begin{bmatrix} V_p^3 \\ V_p^{12} \end{bmatrix} \tag{4.26}$$

The (15x15) order nodal admittance matrix of such a transformer may thus be obtained by expanding eqn. (4.26).

In a similar manner, we can obtain the nodal admittance natrix of a delta/star, three phase/twelve phase transformer shown in Fig. 4.3. Considering the turns ratio of such a transformer to be α_T : 1, its general equation may be written as

$$\begin{bmatrix} I_p^3 \\ \\ -I_p^{12} \end{bmatrix} = \begin{bmatrix} k_1[L][y][J] & -k_2[L][y] \\ \\ -k_3[y][J] & [y] \end{bmatrix} \begin{bmatrix} V_p^3 \\ \\ V_p^{12} \end{bmatrix} \qquad (4.27)$$

The value of the constants are :

For a wye/star transformer :

$$k_1 = \frac{1}{8\alpha_T^2} \; ; \quad k_2 = \frac{1}{8\alpha_T} \; ; \quad k_3 = \frac{1}{\alpha_T} \; ; \qquad J = M$$

For a delta/star transformer :

$$k_1 = \frac{1}{24\alpha_T^2} \; ; \quad k_2 = \frac{1}{8\sqrt{3}\alpha_T} \; ; \quad k_3 = \frac{1}{\sqrt{3}\alpha_T} \; ; \qquad J = K$$

where,

$$K^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Thus we see that the transformer models obtained from eqn. (4.27) are quite simple and reasonably accurate. In a similar manner, we can obtain the models for other types of transformer

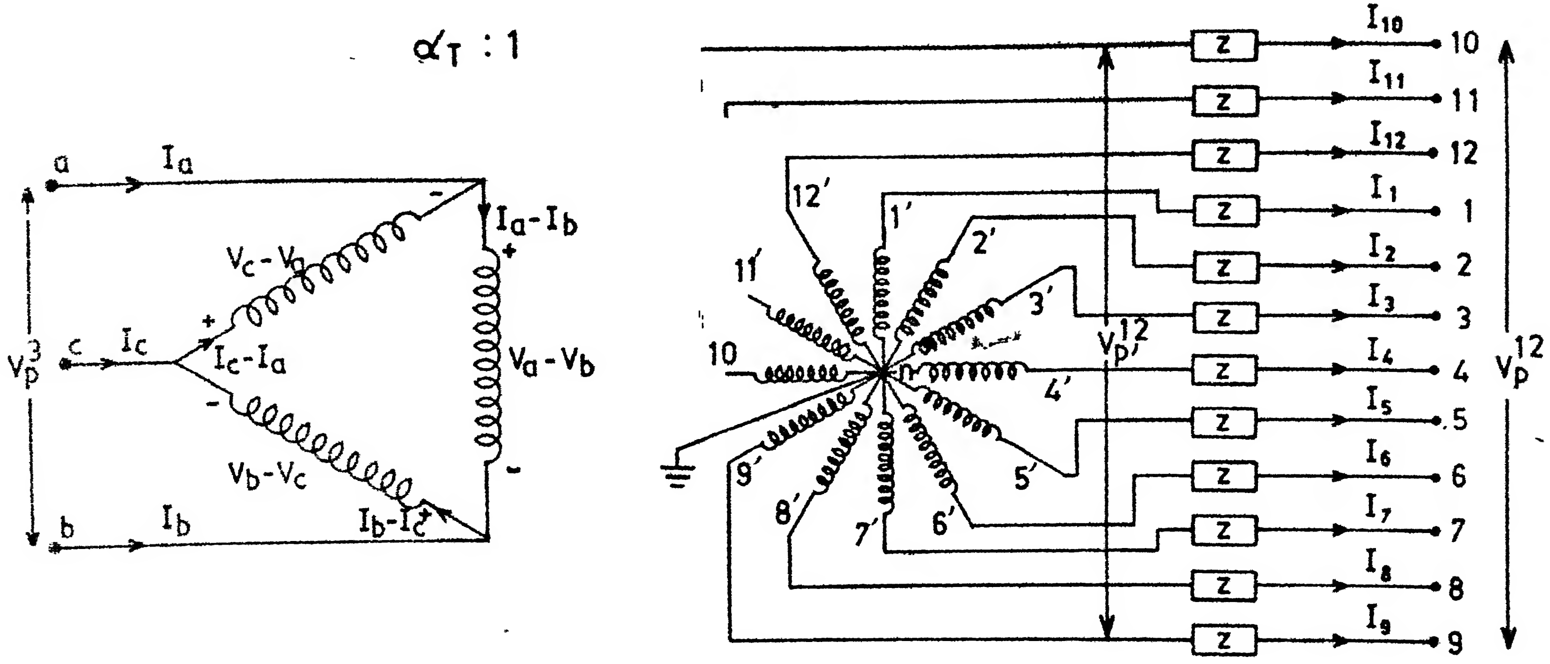


FIG. 4.3 SCHEMATIC REPRESENTATION OF A THREE PHASE / TWELVE PHASE, DELTA/STAR TRANSFORMER WITH TURNS RATIO $\alpha_T : 1$

4.4 MODELLING OF TWELVE PHASE TRANSMISSION LINES

The representation of twelve phase transmission lines in terms of phase impedance matrix, equivalent π-circuit and ABCD parameters has been developed. Three phase equivalent representation of a twelve phase line and vice versa has also been given.

a) Phase Impedance Matrix Representation

A twelve phase short untransposed transmission line possessing cyclic symmetry can be described by its phase impedance matrix as below :

$$Z_p^{12} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1,12} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2,12} \\ & & \vdots & & \\ Z_{12,1} & Z_{12,2} & Z_{12,3} & \cdots & Z_{12,12} \end{bmatrix} \qquad (4.28)$$

The matrix can be fully diagonalized by using symmetrical component twelve phase transformation. However, eqn. (4.28) cannot represent medium and long lines.

b) π-Circuit Representation

The three phase and six phase nominal equivalent π-circuit representation can be extended to twelve phase systems as well in order to represent medium and long lines as shown :

$$\begin{bmatrix} I_{pS}^{12} \\ \\ I_{pR}^{12} \end{bmatrix} = \begin{bmatrix} Y_p^{12} + \frac{1}{2} Y_{sh}^{12} & -Y_p^{12} \\ \\ -Y_p^{12} & Y_p^{12} + \frac{1}{2} Y_{sh}^{12} \end{bmatrix} \begin{bmatrix} V_{pS}^{12} \\ \\ V_{pR}^{12} \end{bmatrix} \tag{4.29}$$

Eqn. (4.29) can be derived by employing the symmetrical lattice equivalent circuit of the twelve phase transmission line as was done for three phase lines. In eqn. (4.29) , the second subscript S and R denote the sending end and receiving end respectively.

c) ABCD Parameters Representation

The twelve phase lines based upon ABCD parameters can be represented as below

$$\begin{bmatrix} V_{pS}^{12} \\ \\ I_{pS}^{12} \end{bmatrix} = \begin{bmatrix} A^{12} & B^{12} \\ \\ C^{12} & D^{12} \end{bmatrix} \begin{bmatrix} V_{pR}^{12} \\ \\ I_{pR}^{12} \end{bmatrix} \tag{4.30}$$

where A^{12},B^{12},C^{12} and D^{12} are square matrices of order 12.

4.5 EQUIVALENT THREE PHASE REPRESENTATION OF A TWELVE PHASE TRANSMISSION LINE

To carry out the analysis of a composite three phase and twelve phase system, equivalent three phase representation of the twelve phase part of it is required. For example, consider a twelve-phase transmission line connected between three-phase buses S and R via three phase/twelve phase transformers as

shown in Fig. 4.4. The phasor equation of the twelve-phase transmission line connected between buses S' and R' in the impedance form is,

$$V_{S'}^{12} - V_{R'}^{12} = Z_p^{12} I_p^{12} \tag{4.31}$$

where Z_p^{12} is the (12x12) phase impedance matrix of the twelve-phase transmission line.

Let the transformers T_1 and T_2 be wye/star, three-phase/twelve-phase of nominal turns ratio. Then, using the voltage relationships of eqn. (4.21), the three phase voltages at the bus S and R may be expressed in terms of the twelve phase voltages $V_{S'}^{12}$ and $V_{R'}^{12}$ and the current I_p^{12} as shown,

$$V_S^3 = \frac{1}{8} L\ V_{S'}^{12} + \frac{1}{8} L\ [z_1]\ I_p^{12} \tag{4.32}$$

$$V_R^3 = \frac{1}{8} L\ V_{R'}^{12} - \frac{1}{8} L\ [z_2]\ I_p^{12} \tag{4.33}$$

where $[z_1]$ and $[z_2]$ are the (12x12) diagonal matrices representing the leakage impedances of the transformers T_1 and T_2 respectively.

Subtracting eqn. (4.33) from eqn. (4.32), we obtain

$$V_S^3 - V_R^3 = \frac{1}{8} L[V_{S'}^{12} - V_{R'}^{12}] + \frac{1}{8} L \left\{[z_1]+[z_2]\right\} I_p^{12}$$

$$= \frac{1}{8} L\, Z_p^{12} I_p^{12} + \frac{1}{8} L \left\{[z_1]+[z_2]\right\} I_p^{12} \tag{4.34}$$

Now using eqn. (4.22), we obtain

$$V_S^3 - V_R^3 = \frac{1}{8} L [Z_p^{12} + z_1 + z_2] M I_p^3 \qquad (4.35)$$

Therefore, $Z_{p,eq}^3 = \frac{1}{8} L[Z_p^{12} + z_1 + z_2]M$ (4.36)

The three-phase equivalent admittance matrix of a twelve-phase line can either be obtained by inverting $Z_{p,eq}^3$ or derived to get

$$Y_{p,eq}^3 = [U + \frac{1}{8} L Y_p^{12} \left\{[Z_1]+[Z_2]\right\} M]^{-1} [\frac{1}{8} LY_p^{12} M] \qquad (4.37)$$

where U is a (3x3) identity matrix.

In a similar, we can obtain the three phase equivalents when other types of transformers are used.

4.6 EQUIVALENT TWELVE-PHASE REPRESENTATION OF A THREE-PHASE LINE

In a composite three-phase and twelve-phase system when the interest of investigation lies in the twelve-phase part of the network, it may be desirable to analyse the system entirely on twelve phase basis. For this purpose, a twelve-phase equivalent of three-phase line is derived. For example, consider a three-phase line connected to the twelve phase buses via twelve-phase/three phase transformers as shown in Fig. 4.5. Th three-phase line is represented by

$$V_{S'}^3 - V_{R'}^3 = Z_p^3 I_p^3 \qquad (4.38)$$

Using relationships (4.21) - (4.23), the voltage drop $V_S^{12} - V_R^{12}$ is expressed as

$$V_S^{12} - V_R^{12} = [\frac{1}{8} M Z_p^3 L + \{ z_1+z_2 \}] I_p^{12} \qquad (4.39)$$

Thus, from eqn. (4.39), the twelve phase equivalent representation of the three phase line is given by

$$Z_{p,eq}^{12} = \frac{1}{8} M Z_p^3 L + [Z_1+Z_2] \qquad (4.40)$$

Similarly, the twelve-phase equivalent admittance matrix is either obtained by inverting $Z_{p,eq}^{12}$ or derived in a similar manner to get

$$Y_{p,eq}^{12} = [U+ \frac{1}{8} M Y_p^3 L \{z_1+z_2\}]^{-1} [\frac{1}{8} M Y_p^3 L] \qquad (4.41)$$

4.7 MODELLING OF TWELVE PHASE LOADS

Twelve-phase loads may be modelled on the basis of constant impedance/admittance to ground. In this case also, we will determine the equivalent three phase representation of a twelve-phase load and vice versa depending upon whether our interest of investigation lies in the three-phase part or the twelve phase part of a composite three-phase, twelve phase system.

Consider a twelve phase load connected via a three-phase/twelve-phase, wye/star transformer to the three-phase bus as shown in Fig. 4.6. For the twelve-phase load, we can write

$$V_p^{12} = Z_L^{12}\ I_p^{12} \tag{4.42}$$

We can also write the voltage V_p^3 as

$$V_p^3 = \frac{1}{8}\ LV_p^{12} + \frac{1}{8}\ L\ [z]\ I_p^{12} \tag{4.43}$$

or

$$V_p^3 = \frac{1}{8}\ LZ_L^{12}MI_p^3 + \frac{1}{8}\ L[z]\ MI_p^3 \tag{4.44}$$

or

$$Z_{L,eq}^3 = \frac{1}{8}\ L\ [Z_L^{12} + z]\ M \tag{4.45}$$

Similarly, we can derive the equivalent three-phase load admittance as

$$Y_{L,eq}^3 = [U + \frac{1}{8}\ L\ Y_L^{12}\ [z]M]^{-1}\ \ [\frac{1}{8}\ L\ Y_L^{12}\ M] \tag{4.46}$$

Now consider a three-phase load connected to a twelve-phase bus through a twelve-phase/three-phase, star/wye transformer as shown in Fig. 4.7. The load can be represented by constant impedance to ground as

$$V_p^3 = Z_L^3\ I_p^3 \tag{4.47}$$

The voltage V_p^{12} can be expressed as

$$V_p^{12} = \frac{1}{8}\ M\ Z_L^3\ LI_p^{12} + [z]\ I_p^{12} \tag{4.48}$$

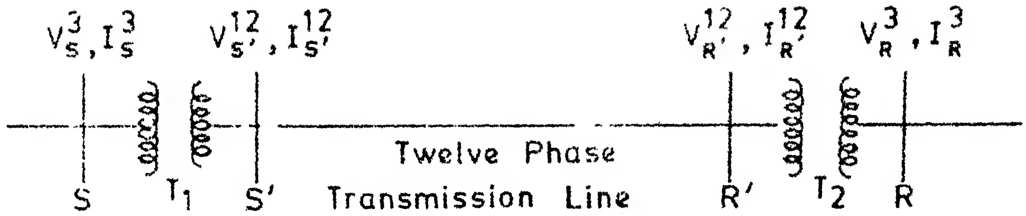


FIG. 4.4 A TWELVE PHASE TRANSMISSION LINE CONNECTED BETWEEN TWO THREE PHASE BUSES VIA THREE PHASE/ TWELVE PHASE TRANSFORMERS T_1 AND T_2

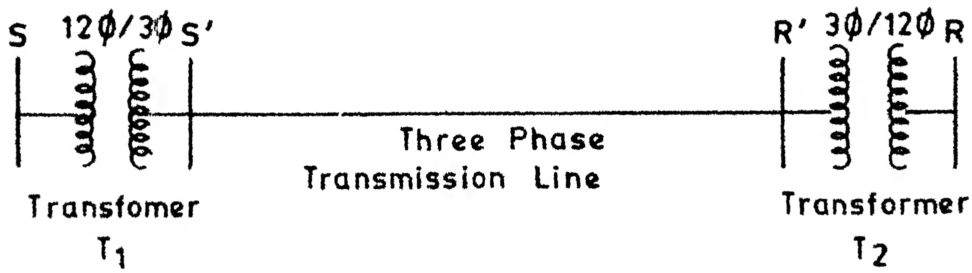


FIG. 4.5 A THREE PHASE TRANSMISSION LINE CONNECTED BETWEEN TWO TWELVE PHASE BUSES THROUGH TWELVE PHASE/THREE PHASE TRANSFORMERS T_1 AND T_2

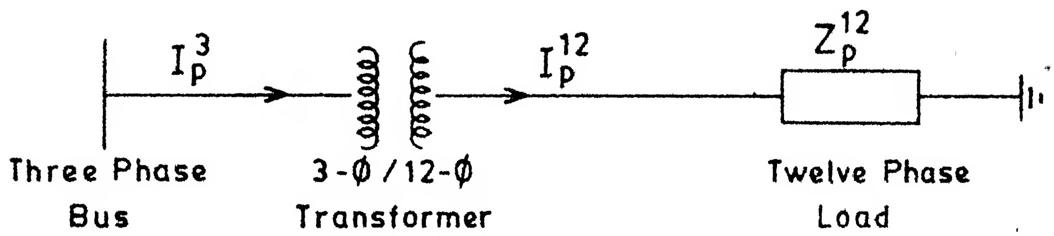


FIG. 4.6 A TWELE PHASE LOAD CONNECTED TO THREE PHASE BUSBAR.

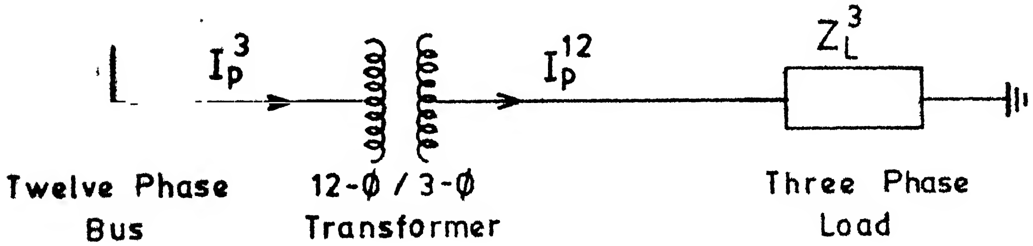


FIG. 4.7 A THREE PHASE LOAD CONNECTED TO TWELVE PHASE BUSBAR

Hence the twelve phase equivalent impedance is

$$Z^{12}_{L,eq} = \frac{1}{8} M Z^{3}_{L} L + Z \tag{4.49}$$

Similarly, the twelve-phase equivalent admittance may be derived as

$$Y^{12}_{L,eq} = [U + \frac{1}{8} M Y^{3}_{L} LZ]^{-1} [\frac{1}{8} M Y^{3}_{L} L] \tag{4.50}$$

4.8 CONCLUSIONS

Although multiphase systems are only of theoretical interest today, twelve-phase transmission system may be feasible if double circuit six phase lines come into existence. This is quite likely to happen in future because of the various advantages of six-phase systems over three-phase systems. In this chapter, the mathematical modelling of various twelve-phase elements such as twelve phase machines, transformers and transmission lines has been given. This modelling may be employed to study the feasibility of twelve-phase systems.

CHAPTER 5

CONCLUSIONS

In the present work, an attempt has been made to study some aspects of mathematical modelling and analysis of multi-phase power systems. Although, multiphase systems are of theoretical interest today, studies have revealed that they may become a reality in future. To get an insight of the behaviour of the multiphase systems, it is necessary to develop the mathematical models of the different components present in it so that detailed analysis like load flow analysis, fault analysis, transient stability analysis, etc. may be conducted.

It has been stressed, that, the mathematical modelling of the elements of a power system in the phase frame of reference gives a realistic picture of the system as it takes into account the inherent unbalances present in it. The detailed models of three phase machines, transformers and transmission lines based upon their symmetrical lattice equivalent circuits have been given.

As a direct extension of three phase mathematical modelling, the representation of elements of multiphase (six phase and twelve phase) power systems in phasor coordinates are given. The equation of the multiphase generators to include the effects of unbalances has been derived in the

matrix form. The multiphase transformers have been modelled based upon their symmetrical lattice equivalent circuits and their nodal relationships. The representation of the transmission line based upon its phase impedance matrix, π-equivalent circuit and ABC -parameters have been given. The multiphase loads are modelled as constant admittances to ground. Equivalent three phase representations of the multiphase transmission lines and vice versa have also been developed.

To show the advantages of the modelling in phasor coordinates, the load flow analysis of a three phase power system network and a mixed three-phase, six-phase power system network was carried out. With unbalances introduced in the three phase power system, it was found that there is a considerable unbalance in the voltage magnitudes and power flows of different phases. The effect of converting an existing double circuits three phase line to a six phase line was studied. It was found, that, the introduction of six phase lines in the system improves the voltage magnitudes at the buses, reduces its angle increases the real power flow and reduces the reactive power flows from the generators. It was also observed, that, the six phase lines tend to reduce slightly the degree of unbalance in the voltage magnitudes of different phases at a bus.

Although sufficient work seems to have been done on the theoretical feasibility of six phase transmission systems, more realistic picture of six phase systems can be had by conducting practical tests on installed six phase transmission lines and the results compared with the theoretical results. Further, work in this area may be pursued by taking a complete six phase system consisting of six phase generators, transformers and transmission lines. The mathematical modelling developed for the twelve phase systems in the present work may be employed to carry out the detailed load flow studies, fault analysis and transient stability analysis of such systems. This will help in getting an overview of the practical feasibility of twelve phase transmission systems which may be a direct consequence of a double circuit six phase line.

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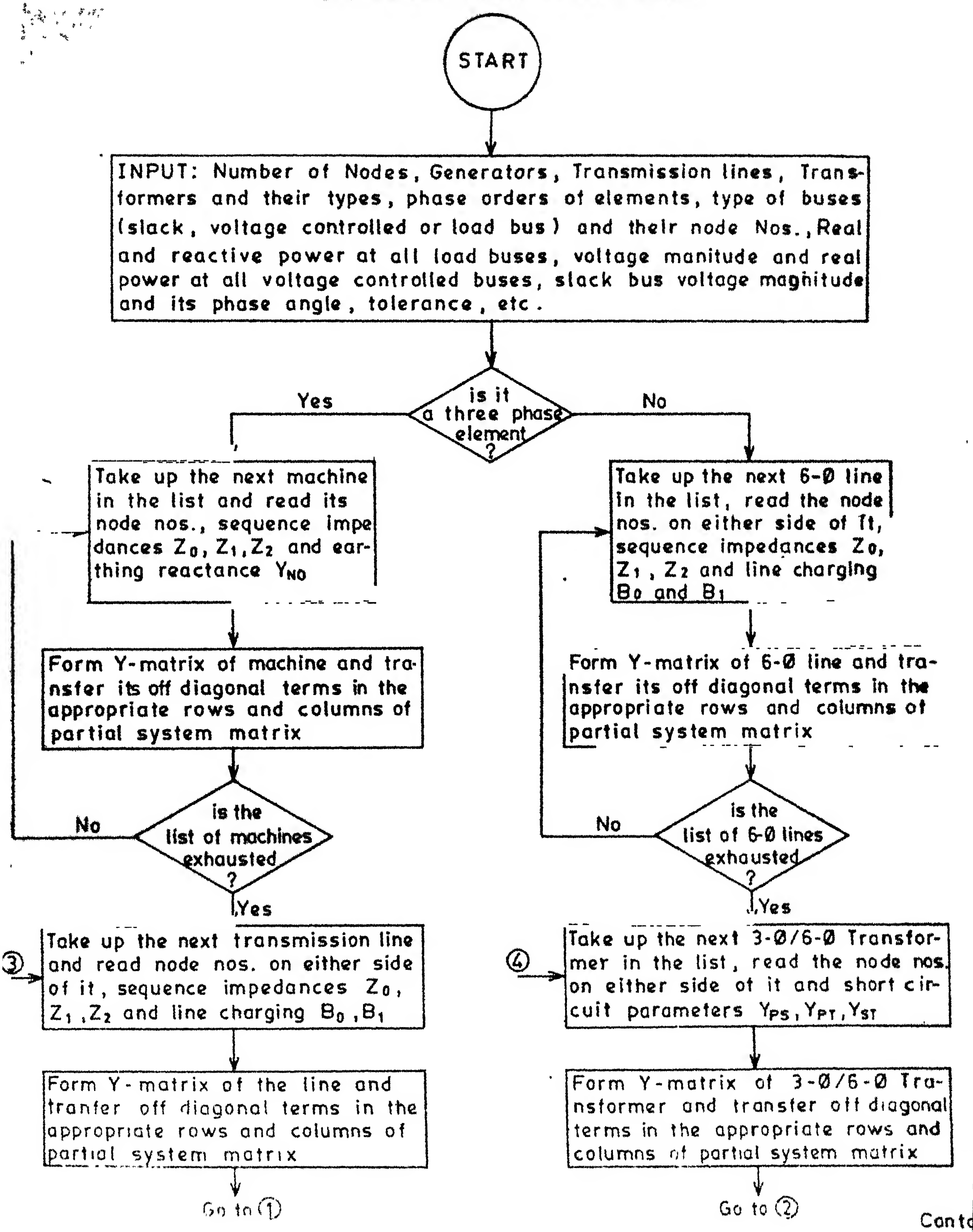
FLOW CHART FOR THREE PHASE AND MIXED THREE PHASE, SIX PHASE LOAD FLOW STUDIES



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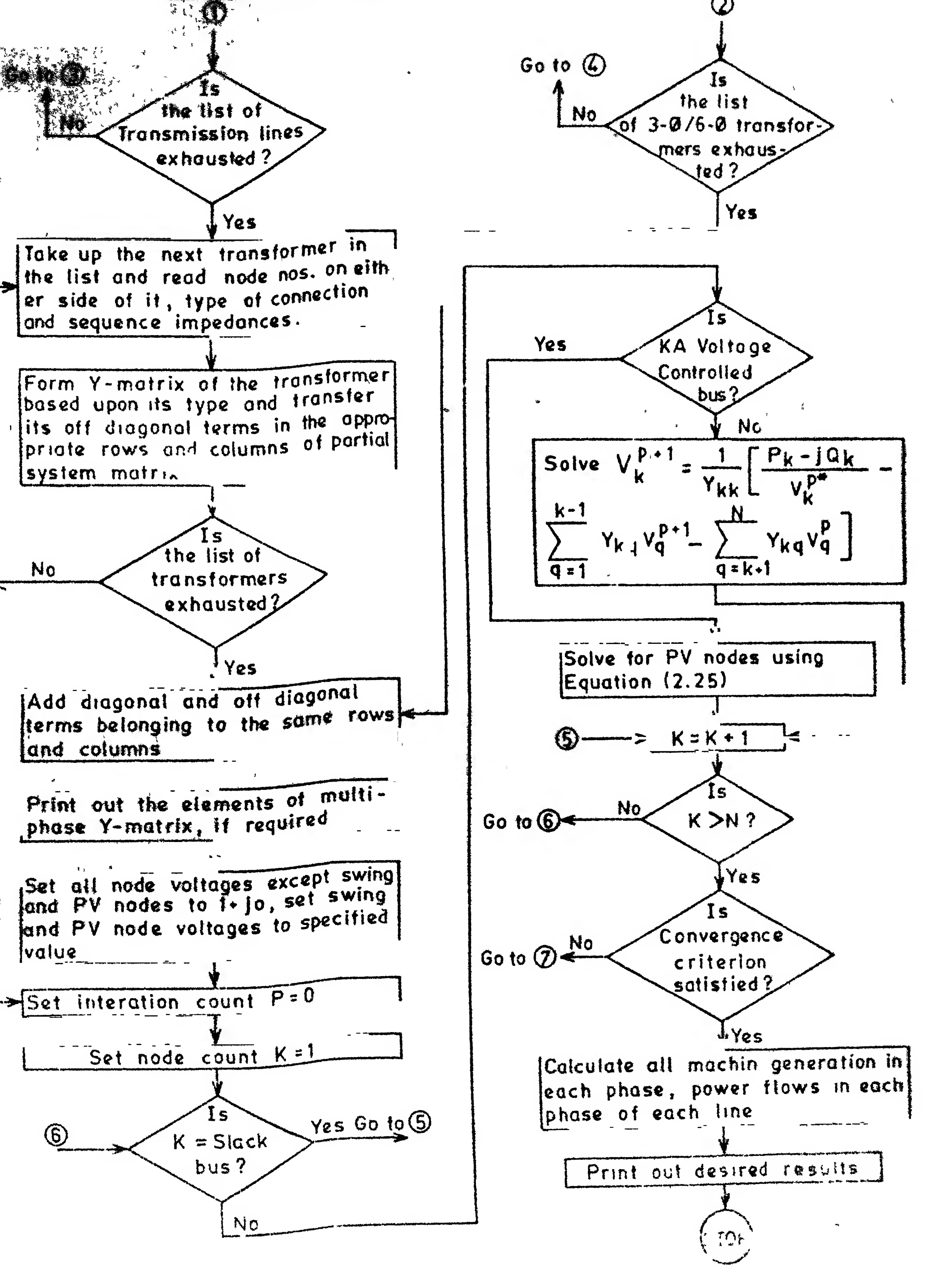
①
Go to ③
No
Is the list of Transmission lines exhausted?
Yes
Take up the next transformer in the list and read node nos. on either side of it, type of connection and sequence impedances.
Form Y-matrix of the transformer based upon its type and transfer its off diagonal terms in the appropriate rows and columns of partial system matrix
Is the list of transformers exhausted?
No
Yes
Add diagonal and off diagonal terms belonging to the same rows and columns
Print out the elements of multi-phase Y-matrix, if required
Set all node voltages except swing and PV nodes to 1+jo, set swing and PV node voltages to specified value
Set interation count P = 0
Set node count K = 1
⑥
Is K = Slack bus?
Yes Go to ⑤
No
②
Go to ④
No
Is the list of 3-Ø/6-Ø transformers exhausted?
Yes
Is KA Voltage Controlled bus?
Yes
No
Solve $V_k^{p+1} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^{p*}} - \sum_{q=1}^{k-1} Y_{kq}V_q^{p+1} - \sum_{q=k+1}^{N} Y_{kq}V_q^{p}\right]$
Solve for PV nodes using Equation (2.25)
⑤
K = K + 1
Is K > N?
No
Go to ⑥
Yes
Is Convergence criterion satisfied?
No
Go to ⑦
Yes
Calculate all machin generation in each phase, power flows in each phase of each line
Print out desired results
TOP

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